## ▶ Setting

The goal of this project is to develop and implement an ingredient of a method for solving a linear system of the form

$$(I \otimes I \otimes A + I \otimes A \otimes I + A \otimes I \otimes I)x = b,$$

where $A$ is a tridiagonal symmetric positive definite matrix of the form

$$A = (n+1)^2 \begin{pmatrix} 2 & -1 & & \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 2 \end{pmatrix} \in \mathbb{R}^{n \times n}.$$

Letting $\mathcal{X}, \mathcal{B} \in \mathbb{R}^{n \times n \times n}$ be such that $x = \mathrm{vec}(\mathcal{X})$ and $b = \mathrm{vec}(\mathcal{B})$, this linear system is equivalent to

$$A \circ_1 \mathcal{X} + A \circ_2 \mathcal{X} + A \circ_3 \mathcal{X} = \mathcal{B}. \tag{1}$$

We will look at two different right-hand sides:

1. $\mathcal{B}_1(i_1, i_2, i_3) = \sin(\xi(i_1) + \xi(i_2) + \xi(i_3))$,
2. $\mathcal{B}_1(i_1, i_2, i_3) = \sqrt{\xi(i_1)^2 + \xi(i_2)^2 + \xi(i_3)^2}$,

with $\xi(i) = i/(n+1)$.

The first step of any low-rank solver for equations of the form (**??**) is to get the right-hand side in compressed format. The goal of this project is to investigate several of the different methods.

## ▶ Tasks

1. Implement the HOSVD and compress $\mathcal{B}_1$ and $\mathcal{B}_2$ for $n = 200$ in the Tucker decompostion, with multilinear ranks chosen such that the norm of the error is bounded by $10^{-4}\|\mathcal{B}_i\|$. Report the obtained multilinear ranks as well as the singular values of the three different matricizations for $\mathcal{B}_1$ and $\mathcal{B}_2$.
2. Formulate a conjecture on the multilinear ranks of $\mathcal{B}_1$ and prove it.
3. Prove that the three different matricizations of $\mathcal{B}_2$ are always identitical.
4. Develop and implement a variant of the HOSVD that uses DEIM or adaptive cross approximation instead of one or all three involved matricizations. Apply it to $\mathcal{B}_1$ and $\mathcal{B}_2$. What is the largest value of $n$ you can handle?

You may use an existing toolbox (e.g., tensor toolbox) for the basic functionality concerning the Tucker decomposition.