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Discussions from: November 5, 2009

Mathematics of Machine Learning

Fall 2009

Assignment Sheet 4

Exercises marked with a $\star$ can be handed in for bonus points. Due date is November 19.

Exercise 1
Show that the set system $\mathscr{F}$ of closed disks in $\mathbb{R}^2$ has $\dim(\mathscr{F}) = 3$.

Exercise 2 ($\star$)
Let $(\mathbb{R}, \mathscr{F})$ be the set system defined by $\mathscr{F} = \{S_p | p \in \mathbb{R}[x]\}$ where $S_p = \{x \in \mathbb{R} | p(x) \geq 0\}$. Show that $\dim(\mathscr{F}) = \infty$.

Exercise 3
In the proof of the ε-net theorem, we consider the events

$$E_S : N \cap S = \emptyset \wedge |M \cap S| \geq k$$

and (for an arbitrary fixed A) the probability $\Pr[E_S \text{ for some } S \in \mathscr{F} \mid A]$. Let $S, S' \in \mathscr{F}$. Show that $E_S = E_{S'}$ whenever $S \cap A = S' \cap A$ and that $\Pr[E_S \text{ for some } S \in \mathscr{F} \mid A] \leq \sum_{S \in \mathscr{F}|_A} \Pr[E_S \mid A]$.

Exercise 4 ($\star$)
Adapt the proof of the ε-net theorem to show the following: there exists a constant C such that for all $(X, \mathscr{F})$, μ, d, and r as in the ε-net theorem, and for all $0 < \delta < \frac{1}{2}$, a random sample of $s = Cr(d \ln r + \ln \frac{1}{\delta})$ points is an ε-net with probability at least $1 - \delta$.

Exercise 5
Suppose you have a biased coin that turns up heads with fixed but unknown probability $0 < p < 1$. Devise a technique by which you can use this biased coin to simulate a completely fair coin. How many coin tosses do you need in expectation?