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Discussions from: November 19, 2009

Mathematics of Machine Learning

Fall 2009

Assignment Sheet 5

Exercises marked with a $\star$ can be handed in for bonus points. Due date is December 3.

Exercise 1
Design a PAC learning algorithm for closed disks in $\mathbb{R}^2$ using linear programming.

Exercise 2 ($\star$)
Let $P_{d,D,k}$ be the system of subsets of $\mathbb{R}^d$ that are defined as the intersection of at most k polynomial inequalties of total degree at most D. That is, for every $S \in P_{d,D,k}$ there exist polynomials $p_1, \dots, p_k \in \mathbb{R}[x_1, \dots, x_d]_D$ such that $S = \{x \in \mathbb{R}^d \mid \forall 1 \leq i \leq k : p_i(x) \leq 0\}$. Give an upper bound on $\dim(P_{d,D,k})$.

Exercise 3
Show that every set of affinely independent points can be shattered using halfspaces.

Note: Recall that a set of points $x_1, \dots, x_k$ is affinely independent if and only if the vectors $(1, x_1^T), \dots, (1, x_k^T)$ are linearly independent.

Exercise 4
Let $\gamma : \mathbb{R} \to \mathbb{R}^n$ be the curve defined by $\gamma(t) := (t^{e_1}, \dots, t^{e_n})^T$ where the e_i are arbitrary pairwise different natural numbers. Show that there exist parameters $t_1, \dots, t_n$ such that $\gamma(t_1), \dots, \gamma(t_n)$ are linearly independent.

Exercise 5 ($\star$)
Recall that $P_{d,D}$ is the system of subsets of $\mathbb{R}^d$ defined by a polynomial inequality of total degree at most D. Show that $\dim(P_{d,D}) \geq \binom{d+D}{d}$.

Hint: Use exercises 3 and 4.