Arolla 2008:
Abstracts of talks

Kasper Andersen: *The isogeny theorem for p-compact groups.*

(Joint work with J. Grodal.)

For algebraic groups, the isogeny theorem states roughly that isogenies between two reductive algebraic groups correspond to isogenies between their root data. This generalizes the isomorphism theorem which encodes the classification of reductive algebraic groups.

In this talk we will show how the classification of p-compact groups generalizes to an isogeny theorem for these. Define an isogeny between connected p-compact groups to be a rational isomorphism. We show that homotopy classes of isogenies between two connected p-compact groups are in 1-1 correspondence with isogenies between their p-adic root data.

Clemens Berger: *The lattice path operad*

(Joint work with M. Batanin.)

We present a coloured operad in sets which permits a unified construction of E_n-operads in monoidal model categories with suitable cosimplicial object. The basic categorical tool consists in a separation of the unary operations of any coloured operad. In particular, our method sheds new light on previous constructions of E_n-operads by Barratt-Eccles, McClure-Smith and Berger-Fresse in the simplicial, topological, and dg- setting. We also recover Tamarkin's construction of a 2-operad action on the category of dg-categories, yielding thereby a "global" proof of the Deligne conjecture on Hochschild cochains.

Natàlia Castellana: *Constructing maps between classifying spaces of p-local finite groups*

(Joint work with A. Libman.)

In this project, we study which is the minimal algebraic data needed to construct maps between classifying spaces of p-local finite groups. More precisely, given a fusion preserving morphism between the corresponding Sylow subgroups, we describe a first approach to the problem of constructing a map between classifying spaces which extends this morphism. We use these techniques to construct permutation representations of p-local finite groups and to describe a notion of index of the Sylow subgroup in the p-local finite group.

Wojciech Chachólski: *How to enumerate Bousfield classes in derived categories of abelian categories?*

(Joint work with J. Scherer and W. Pitsch.)

There are various ways of expressing what it means to be able to do"homotopy theory". A very popular one is using Quillen's model categories. This approach however is not convenient for many purposes: understanding diagram categories, constructing and analyzing homotopy limits and colimits, mapping spaces, unbounded chain complexes etc. The purpose of my talk is to illustrate a modified approach of model approximations. I will show that using this approach all the above problems have a canonical and manageable solutions. I will concentrate on the case of unbounded chain complexes in a rather general abelian category and construct model approximations that corresponds to different choices of weak equivalences. For chain complexes over noetherian rings our model approximations will be enumerated by sets of prime ideals closed under specialization. We will also look at the cases of non Noetherian rings with very few prime ideals but with a rich collection of model approximations and hence a rich collection of Bousfield classes. We aim at explaining classifications of think subcategories in derived categories in model categorical terms.

Ralph Cohen: *Field theories, string topology, and Hochschild homology*

I will begin this lecture by recalling the work of Moore - Segal, and K. Costello on open-closed topological field theories. I will then describe joint work of myself, Andrew Blumberg, and Constantin Teleman, in which our goal is to show how the string topology of a closed oriented manifold M fits into this picture. In particular I will show how, given any closed submanifold $N \subset M$ whose inclusion is 1-connected, the homology of the free loop space arises as the Hochschild cohomology

$$H_*(LM) \cong HH^*(C_*(\mathcal{P}_{N,N}(M)), C_*(\mathcal{P}_{N,N}(M)))$$

where, in general, for two submanifolds $N_0, N_1 \subset M$, $\mathcal{P}_{N_0,N_1}(M))$ consists of paths $\alpha : [0,1] \to M$ with boundary conditions,

$\alpha(0), \in N_0$, $\alpha(1) \in N_1$. The algebra structure when $N_0 = N_1 = N$ is given by the open string topology operations of Sullivan, Harrelson, and Ramirez. Notice that the well known results that

$$HH^*(C_*(\Omega M), C_*(\Omega M)) \cong H_*(LM)$$

and

$$HH^*(C^*(M), C^*(M)) \cong H_*(LM)$$

can be viewed as special cases of this theorem, when one considers the extreme cases of $N = point$, and $N = M$, respectively.

I will also discuss the following more categorical theorem. Let $\mathcal{C}(M)$ be the A_∞- category whose objects are closed submanifolds of M (of any dimension), and whose morphisms between N_0 and N_1 are the singular chains $C_*(\mathcal{P}_{N_0,N_1}(M))$. Composition is again given by the open string topology operations. Then the Hochschild homology of this A_∞-category is the homology of the free loop space,

$$HH_*(\mathcal{C}(M)) \cong H_*(LM).$$

Anand Dessai: *Some applications of elliptic genera to group actions and positive curvature*

Elliptic genera may be viewed as a generalization of classical operators on finite dimensional manifolds (such as the signatur and Dirac operator) to the free loop space. The signature and Dirac operator play a major role in the study of questions concerning group actions and existence of metrics of positive scalar curvature. One expects that elliptic genera play a similar role. In my talk I will survey some applications of elliptic genera to group actions and positive curvature.

Bjørn Ian Dundas: *Witt rings*

The Witt ring is a central ingredient in commutative algebra when lifting from finite to infinite characteristic. Moving into stable homotopy theory, the close analogy, TR, seems to play a similar role when lifting from one chromatic filtration to the next. We have experimental evidence for this through the calculations of Ausoni, Bökstedt, Hesselholt, Madsen and Rognes, but we lack an understanding of the phenomenon equalling the firm foundations in commutative algebra. It has been pointed out that classes determining higher chromatic phenomena appear in the building blocks for TR explaining the observed “redshift”, but this is more by accident than by design. Can one build Witt rings that exactly pin down the step from one chromatic level to the next?

In the talk I will try to sum up what is known and do some speculations as to how Witt rings can play a systematic role in stable homotopy theory. The presented material relies on joint work in progress and conversations with among others Baker, Bruner, Brun, Carlsson, Douglas and Rognes.

Benoît Fresse: *Koszul duality of the chain little cubes operads*

The bar duality of operads shows that any chain operad P has a cofibrant model given by a cobar construction $B^c(D)$ on a cooperad D associated to P. For certain good operads, the Koszul operads, the homotopy type of the cooperad D can be determined explicitly by using the Koszul duality of operads. As an example, the operad of commutative algebras $\mathtt{C}$ is Koszul and its dual is the cooperad of Lie coalgebras $\mathtt{L}^\vee$.

The determination of the cooperad D is important in the study of the homotopy category of P-algebras. For instance, in the case of the commutative operad $\mathtt{C}$, we recover the duality of rational homotopy between commutative and Lie algebras.

The first objective of this talk is to prove that the chain operads of little n-cubes $\mathtt{E}_n$ have a cofibrant model of the form $B^c(\Lambda^{-n}\mathtt{E}_n^\vee)$, where $D = \Lambda^{-n}\mathtt{E}_n^\vee$ is the operadic n-fold desuspension of the cooperad $\mathtt{E}_n^\vee$ dual to $\mathtt{E}_n$ in $\mathbb{k}$-modules.

Usual models of little n-cubes operads form a nested sequence of chain operads

$$\mathtt{E}_1 \subset \mathtt{E}_2 \subset \cdots \subset \operatorname*{colim}_n \mathtt{E}_n = \mathtt{E},$$

where $\mathtt{E}$ is an E_∞-operad. In a previous work (joint work with C. Berger), we made explicit an operad morphism $\sigma : \mathtt{E} \to \Lambda^{-1}\mathtt{E}$ for a particular E_∞-operad $\mathtt{E}$, the Barratt-Eccles operad. This morphism determines a natural action of $\mathtt{E}$ on the suspension of $\mathtt{E}$-algebras (where we consider the genuine suspension in chain complexes). Surprisingly, we have observed that σ maps $\mathtt{E}_n$ to $\Lambda^{-1}\mathtt{E}_{n-1}$ and induces an operad morphism $\sigma : \mathtt{E}_n \to \Lambda^{-1}\mathtt{E}_{n-1}$. In the talk, we shall prove that the induced morphisms on the cobar construction $B^c(\Lambda^{-n}\sigma^\vee) : B^c(\Lambda^{1-n}\mathtt{E}_{n-1}^\vee) \to B^c(\Lambda^{-n}\mathtt{E}_n^\vee)$ fit a commutative diagram

$$\begin{array}{ccccccccc}
B^c(\Lambda^{-1}\mathtt{E}_1^\vee) & \dashrightarrow & \cdots & \dashrightarrow & B^c(\Lambda^{1-n}\mathtt{E}_{n-1}^\vee) & \dashrightarrow & B^c(\Lambda^{-n}\mathtt{E}_n^\vee) & \dashrightarrow & \cdots \\
\sim\downarrow & & & & \sim\downarrow & & \sim\downarrow & & \\
\mathtt{E}_1 & \hookrightarrow & \cdots & \hookrightarrow & \mathtt{E}_{n-1} & \hookrightarrow & \mathtt{E}_n & \hookrightarrow & \cdots
\end{array}$$

and, hence, represent the operad embeddings $\iota : \mathtt{E}_{n-1} \hookrightarrow \mathtt{E}_n$ at the level of cofibrant models.

The Quillen homology of $\mathtt{E}_n$-algebras $H_*^{E_n}(A)$ is defined as the derived functor of indecomposables from $\mathtt{E}_n$-algebras to chain complexes. We use our result and the Koszul duality theory of operads to give the form of a chain complex that computes $H_*^{E_n}(C^*(S^m))$, where $C^*(S^m)$ is the cochain algebra of an m-sphere S^m. Our motivation comes from researches on the iterated bar construction: for the cochain algebra of a space X, we proved that $H_*^{E_n}(C^*(X))$ agrees with the cohomology of an iterated bar complex $B^n(C^*(X))$ and determines the profinite cohomology of $\Omega^n X$, the nth iterated loop space of X.

Søren Galatius: *Spaces of graphs*

I will discuss the homotopy type of various spaces of graphs in Euclidean space. This is part of a calculation (math/0610216) of the homology of $\mathrm{Aut}(F_n)$ in the "stable range". Here F_n is a free group on n generators and $\mathrm{Aut}(F_n)$ is its automorphism group. Hatcher and Vogtmann established a stable range: $H_k(Aut(F_n))$ is independent of n as long as $n > 2k + 1$.

Tom Goodwillie: *Functor calculus and pseudoisotopy spaces*

Let $F(M)$ be the homotopy fiber of the suspension map from pseudoisotopies of M to pseudoisotopies of $M \times I$. We show that when a handle of index $k \geq 3$ is attached to M the induced map from $F(M)$ to $F(M \cup H)$ is approximately $(dim(M) + k)$-connected. The proof of this relative connectivity result uses multirelative connectivity (connectivity of cubical diagrams) in an essential way, and the broader purpose of the talk will be to illustrate some ways of using "analyticity" in functor calculus. If time permits, we will go a little further than the stated result, bringing in Weiss' orthogonal calculus.

Ian Hambleton: *Equivariant CW-Complexes and the orbit category*

(Joint work with Semra Pamuk and Ergun Yalcin.)

In the talk I will describe a general framework for constructing G-CW complexes via the orbit category, based on work of tom Dieck and Lueck. We further develop homological algebra over the orbit category by analogy with integral representations of finite groups and obtain a generalization of a theorem of Rim (Annals, 1959) to this setting. As an application, we show that the symmetric group $G = S_5$ admits a finite G-CW complex X homotopy equivalent to a sphere, with cyclic isotropy subgroups. This is the first non-trivial case of an open problem: which finite groups of rank 2 can act on spheres with rank 1 isotropy ?

Jean-Claude Hausmann: *Conjugation spaces*

There are classical examples of spaces X with an involution τ whose mod 2-comhomology ring ressembles that of their fixed point set X^τ: there is a ring isomorphism $\kappa : H^{2*}(X) \approx H^*(X^\tau)$. Such examples include complex Grassmannians, toric manifolds, polygon spaces, etc. In this talk, we show that, for the above examples and many others, the ring isomorphism κ is part of an interesting structure in equivariant cohomology called an H^*-*frame*. A space with involution admitting an H^*frame is called a *conjugation space*. Many examples of conjugation spaces can be constructed.

An H^*frame, if it exists, is natural and unique and enjoys many properties. For instance, for a conjugate-equivariant complex vector bundle ("real bundle" in the sense of Atiyah) over a conjugation space, we show that the isomorphism κ sends its Chern classes onto the Stiefel-Whitney classes of its fixed bundle. Also, the isomorphism κ commutes with the Steenrod operations [Franz-Puppe].

This talk is based on a joint paper with Tara Holm and Volke Puppe, extended later by Matthias Franz and Volker Puppe. Results of Martin Olbermann will be discussed as well as recent examples of Wolfgang Pitsch and Jérôme Scherer.

Lars Hesselholt: *K-theory of the dual numbers*

(Joint work with Vigleik Angeltveit and Teena Gerhardt.)

Long ago, Soulé showed that the groups $K_n(\mathbb{Z}[x]/(x^2),(x))$ are finitely generated abelian groups and that their rank is 0, for n even, and 1, for n odd. We show that, for n even, the finite group $K_n(\mathbb{Z}[x]/(x^2),(x))$ has order

$$|K_n(\mathbb{Z}[x]/(x^2),(x))| = n\,!$$

and that, for n and odd, the group $K_n(\mathbb{Z}[x]/(x^2),(x))$ is a free abelian group of rank 1. We determine the structure of the even groups in low degrees.

Nick Kuhn: *Towards calculating the unstable v_n-periodic homotopy groups of spheres*

Much of what we know about the unstable homotopy groups of spheres comes from our detailed understanding of the v_1-periodic part, whose study goes back to the work of Adams in the 1960's. We discuss how a number of tools are in place, and fit well together, to begin a similar analysis of the v_n-periodic part, for $n > 1$. These tools include: (a) telescopic functors Φ_n from spaces to spectra, constructed by Bousfield and me, (b) an analysis of the effect of Φ_n (Hopf invariants) in height

n complex oriented theories, implicit in Rezk's work, (c) the Goodwillie resolution of odd spheres as analyzed by Arone and Mahowald, and (d) when the sphere is S^1, a conjectural connection between this resolution and the 'Whitehead conjecture' resolution. One punchline is that we may soon be able to make explicit calculations of $E_n^*(\Phi_n(S^d))$, where E_n is the nth p-adic integral Morava K-theory.

Pascal Lambrechts: *Rational homology of the space of smooth embeddings*

This is joint work with Greg Arone and Ismar Volic. We consider (a variation of) the space of smooth embeddings $Emb(M, R^d)$, of a compact manifold M in a large euclidean space is an invariant of the rational homotopy type of M. For example the spaces $Emb(S^3 \times S^3, R^n)$ and $Emb(RP(3) \times S^3 \# RP(6))$ have the same rational homology and this homology is highly non trivial. A special case of this is when M is 1-dimensional where we get that the homology of this embedding space is the homology of an explicit graph complex (also joint work with V. Turchin). The techniques are Goodwillie-Weiss calculus of embeddings, Weiss orthogonal calculus, and a relative version of Kontsevich's formality of the little disks operad.

Assaf Libman: *The Burnside ring of a saturated fusion system*

The Burnside ring of a finite group G is the Grothendieck ring of the set of isomorphism classes of finite G-sets with disjoint union and product giving rise to the ring structure. This is an important invariant of G which has a fundamental importance in the study of Mackey functors over G. In this talk I will construct the Burnside ring of saturated fusion systems. I will show that after p-localisation, the Burnside ring of a fusion system which is associated with a finite group G is a section of the Burnside ring of G. I will also discuss the algebraic properties of the p-local Burnside ring of a fusion system. This is joint work with Antonio Diaz.

Jacob Lurie: *Moduli problems for ring spectra*

A result of Hinich asserts that for any "moduli space" M equipped with a base point x, a formal neighborhood of x in M can be completely described by a certain differential graded Lie algebra. In this talk, I will explain Hinich's result in more detail, and give a generalization to noncommutative geometry.

Holger Reich: *Algebraic K-theory of virtually cyclic groups*

The talk will report on joint work with Jim Davis and Frank Quinn. The family in the K-theoretic Farrell-Jones conjecture can be reduced to only those virtually cyclic groups which admit a surjection onto a cyclic group. I will try to explain a proof of this fact that uses geometric algebra.

Birgit Richter: *An involution on the K-theory on (some) bimonoidal categories*

On every bimonoidal category with anti-involution, R, there is an involution on the associated K-theory. This K-theory is the algebraic K-theory of the spectrum associated to R. In the talk I will give the construction of this involution and present examples such as the K-theory of rings with anti-involution and Waldhausen's A-theory of spaces of the form BBG for an abelian group G.

Paolo Salvatore: *The topological cyclic Deligne conjecture*

I show that the totalization of a topological cyclic operad with multiplication has an action of the framed little 2-discs operad. I first reinterpret geometrically the non-cyclic version of the conjecture by McClure and Smith in terms of cacti operads. This leads naturally to the cyclic version. The main application is about spaces of embeddings, when the cyclic operad is the framed little n-discs operad.

Christian Schlichtkrull: *Higher topological Hochschild homology of Thom spectra*

We analyze the higher topological Hochschild homology of Thom spectra in general and we explain its relationship to free torus mapping spaces. We also comment on the relationship to topological Andre-Quillen homology and higher topological cyclic homology.

Ulrike Tillmann: *From braid to mapping class groups, configuration to moduli spaces*

Dehn twists around simple closed curves in oriented surfaces satisfy the braid relations. This gives rise to a group theoretic map from braids to mapping class groups. In my talk I will present joint work with Graeme Segal in which we prove an old conjecture of Harer (previously proved in joint work with Song) concerning the induced map in stable homology. Based on a geometric interpretation of the group homomorphism the proof also yields results for the unstable homology.

Ismar Volic: *Embedding calculus and Milnor invariants*

We will describe how embedding calculus of functors can be used for classifying Milnor invariants of classical homotopy string links and placing them into a homotopy-theoretic framework via a certain cosimplicial model. This model keeps track all finite type, or Vassiliev, invariants of string links of which Milnor invariants are examples. We will also discuss how this approach can be generalized for defining analogs of Milnor invariants for homotopy links of spheres of any dimension in $\mathbb{R}^{\Bbbk}$, where the theory connects with work of Koschorke. This is joint work with Brian Munson.

Nathalie Wahl: *Homological conformal field theories*

A conformal field theory is a monoidal functor from the cobordism category of closed 1-manifolds with morphisms the chain on the moduli spaces of Riemann cobordisms between the 1-manifolds. A homological field theory is the homological shadow of such a functor. Examples of such theories have been constructed on the homology of certain free loop spaces, and on the hochschild homology of Frobenius algebras. This talk will be an attempt to explain why a large part of this very rich structure of homological conformal field theory often seems to be trivial.