

# Algebraic $K$ -theory

## Spring 2011

### Syllabus

1. Introduction: motivations and relations with other fields [Week 1]
2.  $K_0$  and classification of modules
  - (a) Definition and elementary properties of  $K_0$  [Weeks 1 to 5]
    - i. Group completion
    - ii. Grothendieck groups
    - iii. Devissage
    - iv. The Resolution Theorem
    - v. Stability
    - vi. Multiplicative structure
  - (b) Functoriality of  $K_0$  [Weeks 6 to 8]
    - i. Exact functors
    - ii. Naturality of  $K_0(R)$
    - iii. Localization
3.  $K_1$  and classification of invertible matrices [Weeks 9 to 11]
  - (a) Elementary matrices and commutators
  - (b) Definition and elementary properties of  $K_1$
  - (c) Generalized determinants
  - (d)  $K_1$  as a Grothendieck group
4.  $K_2$  and relations among matrices
  - (a) Definition and elementary properties of  $K_2$  [Week 12]
  - (b) Exact sequences [Weeks 13]
    - i. The relative sequence
    - ii. Excision and the Mayer-Vietoris sequence
    - iii. The localization sequence
  - (c) Matsumoto's Theorem [Week 14]

## **Bibliography**

This course will be based primarily on Chapters 3, 4, 5, 6, 9, 12, 13, and 14 of the following text.

Bruce A. Magurn *An Algebraic Introduction to K-theory*, Encyclopedia of Mathematics and its Applications **87**, Cambridge University Press, 2009.