Elements of Homotopy **Fall 2008**

Prof. Kathryn Hess

Series 6

Exercise 1.

(a) Construct a space Y such that $\pi_1(Y) \cong \mathbb{Z}/3\mathbb{Z}$.

(b) For each $n \geq 2$, construct a space Y such that $\pi_1(Y) \cong \mathbb{Z}/n\mathbb{Z}$.

Note that $\mathbb{Z}/n\mathbb{Z} \cong F(a)/\sim$ such that $\sim$ is generated by the relation $a^n \sim 1$.

Exercise 2. Let $n \geq 2$.

(a) Construct a space Y such that $\pi_1(Y) \cong D_n$.

Recall that the dihedral group $D_n \cong F(a,b)/\sim$ such that $\sim$ is generated by the relations $a^n \sim 1$, $b^2 \sim 1$, and $bab \sim a^{-1}$.

Exercise 3. Let G be a group.

(a) Prove that $G \cong F(\{a_\alpha\})/\sim$ for some set of symbols $\{a_\alpha\}$ such that $\sim$ is generated by a set of relations $\{w_\beta \sim 1\}$ and each w_β is a word in the symbols $\{a_\alpha\}$.

(b) Assuming the set of relations $\{w_\beta \sim 1\}$ is finite, construct a space Y such that $\pi_1(Y) \cong G$.

(c) Generalize the theorem in class on attaching a disk D^2 by its boundary S^1 to a space X, to allow for attaching a set of disks $\{D^2\}_\beta$.

(d) For an arbitrary group G, construct a space Y such that $\pi_1(Y) \cong G$.

Here are some references for this material: [1, Section 1.2].

References

[1] Allen Hatcher. *Algebraic topology*. Cambridge University Press, Cambridge, 2002.