Higher Algebraic K-theory
Exercise Set 11

15.12.2011

1. Apply Dévissage to proving that if p is any prime number and r is any positive integer, then

$$K_n\big(\mathscr{M}(\mathbb{Z}/p^r\mathbb{Z}), \mathscr{E}\big) \cong K_n(\mathbb{Z}/p\mathbb{Z})$$

   for all $n \geq 0$, where $\mathscr{E}$ is the usual class of exact sequences of finitely generated modules.

2. Let m be any positive integer. Prove that if $m = p_1^{r_1} p_2^{r_2} \cdots p_k^{r_k}$, where $p_1, ..., p_k$ are distinct primes and $r_i \geq 1$ for all i, then

$$K_n\big(\mathscr{M}(\mathbb{Z}/m\mathbb{Z}), \mathscr{E}\big) \cong \bigoplus_{i=1}^{n} K_n(\mathbb{Z}/p_i\mathbb{Z})$$

   for all $n \geq 0$.

3. Let $\mathscr{T}$ denote the category of finitely generated abelian groups that have no free part, i.e., the category of finitely generated torsion abelian groups. Prove that

$$K_n(\mathscr{T}, \mathscr{E}) \cong \bigoplus_{p \text{ prime}} K_n(\mathbb{Z}/p\mathbb{Z})$$

   for all $n \geq 0$.

   Hint: Observe that $\mathscr{T} = \bigcup_{m \geq 2} \mathscr{T}_m$, where $\mathscr{T}_m$ denotes the full subcategory of $\mathscr{T}$ determined by the finitely generated, abelian m-torsion groups. Deduce that

$$K_n(\mathscr{T}, \mathscr{E}) \cong \sum_{m \geq 2} K_n(\mathscr{T}_m, \mathscr{E}),$$

   then apply exercise 2.