1. Let $K_{\bullet,\bullet}$ be a bisimplicial set. Prove that there is a natural isomorphism
\[
\text{diag}(K_{\bullet,\bullet}) \cong |K_{\bullet,\bullet}|_{\text{simp}}
\]
of simplicial sets.

2. Let $K_\bullet$ and $L_\bullet$ be simplicial sets. Let $(K \times L)_{\bullet,\bullet}$ denote the bimultiplicial set with $(K \times L)_{m,n} = K_m \times L_n$ and the obvious horizontal and vertical faces and degeneracies. Prove that $|((K \times L)_{\bullet,\bullet})| \cong |K_\bullet| \times |L_\bullet|$. What can you say about $|((K \times L)_{\bullet,\bullet})|_{\text{simp}}$?

3. Show that the diagonal functor $\text{diag} : \text{bi-sSet} \rightarrow \text{sSet}$ admits a left adjoint.

4. Let $\mathcal{C}_\bullet$ be a simplicial category, which can be seen as functor $\Delta^{op} \rightarrow \text{Cat}$ or as $\{\{\mathcal{C}_n\}_{n}, \{d_i\}_i, \{s_j\}_j\}$, a graded category endowed with face and degeneracy functors satisfying the simplicial identities.

Postcomposition with the nerve functor defines a functor
\[
\text{sN}_\bullet : \text{sCat} \rightarrow \text{bi-set}.
\]
Moreover, there is a functor
\[
\Phi : \text{Cat} \rightarrow \text{sCat}
\]
given by $\Phi(\mathcal{C}) = \text{Fun}(\iota(-), \mathcal{C}) : \Delta^{op} \rightarrow \text{Cat}$, where $\iota : \Delta \rightarrow \text{Cat}$ is the functor of Exercise 2(a) of Exercise set 3, and $\text{Fun}(\mathcal{D}, \mathcal{C})$ denotes the category of functors from $\mathcal{D}$ to $\mathcal{C}$. For any category $\mathcal{C}$, explain the relationship between the spaces $|\mathcal{C}|$ and $|\text{sN}_\bullet \circ \Phi(\mathcal{C})|$.  

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