1 Simplicial suspension

Define the simplicial suspension $E K \bullet$ of a simplicial set $K \bullet$ as follows:
$E_0(K) = b_0$ and $E_n(K)$ is the set of pairs $(i, x)$, where $i \geq 1$ is an integer and $x \in K_{n-i}$.

The face and degeneracy operators are generated by

- $d_0(1, x) = b_n$ for all $x \in K_n$,
- $d_1(1, x) = b_0$ for all $x \in K_0$,
- $d_{i+1}(1, x) = (1, d_i x)$ for all $x \in K_n, n > 0$,
- $s_0(i, x) = (i + 1, x)$ and
- $s_{i+1}(1, x) = (1, s_i x)$,

with all other face and degeneracy maps defined by the requirement that $E K \bullet$ be a simplicial set.

Show that $H_* K \bullet \cong H_{*+1} E K \bullet$.

2 Group homomorphisms and simplicial maps

Show that a group homomorphism $\varphi : G \longrightarrow H$ induces a simplicial map $B_* \varphi : B_* G \longrightarrow B_* H$.

3 Group homomorphisms and simplicial homotopies

Let $\varphi, \psi \in Gr(G, H)$.

1. Under which condition is $B_* \varphi \simeq B_* \psi$?
2. Give examples!
4 Natural transformations and simplicial homotopies

Let $C, D$ be categories and $F, G$ functors $C \to D$. Show that a natural transformation $F \Rightarrow G$ induces a simplicial homotopy $N\bullet F \simeq N\bullet G$ in the corresponding nerves of $C$ and $D$.

5 $\Delta [n]$ is contractible

Show that $\text{Id}_{\Delta[n]} \simeq e \ \forall n$, where $e : \Delta [n] \to \Delta [n] : x \mapsto (0_0 \ldots 0_m) \ \forall x \in \Delta [n]_m$.

6 The circle and homology

Give a nontrivial simplicial map $\varphi : \partial \Delta [2] \to \Delta [1] / \partial \Delta [1]$ and show that it induces isomorphisms $H_n \varphi : H_n(\partial \Delta [2]) \to H_n(\Delta [1] / \partial \Delta [1])$ for all $n$. 