1. Apply the Seifert-van Kampen Theorem to establishing the following isomorphisms.

(a) \( \pi_1(S^1 \vee \cdots \vee S^1, 1) \cong \mathbb{Z} \ast \cdots \ast \mathbb{Z} \).

(b) \( \pi_1(S^n, z_0) \cong \{0\} \) for all \( z_0 \in S^n \) and all \( n \geq 2 \). Why can’t we use Seifert-van Kampen to conclude that \( \pi_1(S^1, 1) \) is trivial as well?

(c) If \( f : (S^1, 1) \to (X, x_0) \) is a based continuous map, then

\[
\pi_1(X \cup_f D^2, x_0) \cong \pi_1(X, x_0)/N,
\]

where \( N \) is the normal subgroup generated by \( \{[f]_*\} \).

2. For any positive integer \( n \), explain how to construct a path-connected space \( X_n \) from a circle and a disk such that \( \pi_1(X_n, x_0) \cong \mathbb{Z}/n\mathbb{Z} \). Show that \( X_2 \) is homeomorphic to \( \mathbb{R}P^2 \), the real projective plane, which is usually defined as the quotient of \( D^2 \) by the relation that identifies antipodal points on its boundary.