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Discussions: March 03, 2009

Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 2

Exercise 1

A factory produces two different products. To create one unit of product 1, it needs one unit of raw material A and one unit of raw material B. To create one unit of product 2, it needs one units of raw material B and two units of raw material C. Raw material B needs preprocessing before it can be used, which takes one minute per unit. At most 20 hours of time is available per day for the preprocessing. Raw materials of capacity at most 1200 can be delivered to the factory per day. One unit of raw material A, B and C has size 4, 3 and 2 respectively.

At most 130 units of the first and 100 units of the second product can be sold per day. The first product sells for 6 CHF per unit and the second one for 9 CHF per unit.

1. Formulate the problem of maximizing turnover as a linear program and solve it using Zimpl and an LP solver of your choice.
2. Can you formulate the program as a linear program with two variables as well?

Exercise 2

Show that the set of feasible solutions of every linear program is convex.

Exercise 3

Provide an example of a convex and closed set $K \subseteq \mathbb{R}^2$ and a linear objective function $c^T x$ such that $\inf\{c^T x \ : \ x \in K\} > -\infty$ but there does not exists an $x^* \in K$ with $c^T x^* \leq c^T x$ for all $x \in K$.

Exercise 4

Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a linear map and let $K \subseteq \mathbb{R}^n$ be a set.

1. Show that $f(K) := \{f(x) : x \in K\}$ is convex if K is convex. Is the reverse also true?
2. Prove that $\text{conv}(f(K)) = f(\text{conv}(K))$.

Exercise 5

Let $X \subseteq \mathbb{R}^n$ be a set of points. Prove the following statements:

1. The set
$$\text{cone}(X) := \left\{ \sum_{i=1}^{t} \lambda_i x_i \ : \ t \in \mathbb{N}, x_i \in X, \lambda_i \geq 0 \forall i = 1, \ldots, t \right\}$$
is a cone.
2. Each cone containing X also contains $\text{cone}(X)$.