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Due Date: March 16, 2009
Discussions: March 17, 2009

Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 4

Exercise 1

A company must deliver d_i units of its product at the end of the ith month. Material produced during a month can be delivered either at the end of the same month or can be stored as inventory and delivered at the end of a subsequent month; however, there is a storage cost of c_1 dollars per month for each unit of product held in inventory. The year begins with zero inventory. If the company produces x_{i-1} units in month $i-1$ and x_i units in month i, it incurs a cost of $c_2 \cdot |x_i - x_{i-1}|$ dollars, reflecting the cost of switching to a new production level. Formulate a linear programming problem whose objective is to minimize the total cost of the production and inventory schedule over a period of twelve months. Assume that inventory left at the end of the year has no value and does not incur any storage costs.

Exercise 2

The vector $x^* = (0,1,1,1)$ is an optimal solution of

$$\begin{aligned} \min\ & (1,1,0,2) \cdot x \\ \underbrace{\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 2 \\ 2 & 0 & 0 & 0 \end{pmatrix}}_{=A} x &= \begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix} \\ x &\geq \mathbf{0} \end{aligned}$$

with $x = (x_1, x_2, x_3, x_4) \in \mathbb{R}^4$. Use the proof of Lemma 3.1 to find another optimal solution x' such that $A_{J'}$ has full column rank with $J' = \{i \mid x'_i > 0\}$.

Exercise 3

Solve the following tableau with the simplex algorithm:

x_1	x_2	x_3	x_4	x_5	
-3	-2	-1	0	0	0
1	2	3	1	0	3
1	-1	2	0	1	2

For each iteration give the simplex tableau, the current basis and the indices leaving/entering the basis.

Exercise 4

Solve the following linear program by using the simplex method

$$\begin{aligned} \min \begin{pmatrix} -4 & -1 & 1 & 0 & 0 & 0 \end{pmatrix}^T x & \\ \begin{pmatrix} 1 & 0 & 3 & 1 & 0 & 0 \\ 3 & 1 & 3 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} x &= \begin{pmatrix} 6 \\ 9 \\ 2 \end{pmatrix} \\ x &\geq \mathbf{0} \end{aligned}$$

Start with the basis $B = \{4,5,6\}$. For each iteration give the simplex tableau and the indices leaving/entering the basis.

Exercise 5
Suppose you are given an oracle algorithm, which for a given polyhedron $P = \{\tilde{x} \in \mathbb{R}^{\tilde{n}} : \tilde{A}\tilde{x} \leq \tilde{b}\}$ gives you a feasible solution or asserts that there is none. Show that using a single call of this oracle one can obtain an optimum solution for the LP

$$\min\{c^T x : x \in \mathbb{R}^n;\ Ax = b;\ x \geq \mathbf{0}\},$$

assuming that the LP is feasible and bounded.
Hint: Use duality.