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Due Date: March 23, 2009
Discussions: March 24, 2009

Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 5

Exercise 1

A manager of an oil refinery has 8 million barrels of crude oil A and 5 million barrels of crude oil B allocated for production during the coming month.

These resources can be used to make either gasoline, which sells for 38$ per barrel, or home heating oil, which sells for 33$ per barrel. There are three production processes with the following characteristics:

	Process 1	Process 2	Process 3
Input crude A	3	1	5
Input crude B	5	1	3
Output gasoline	4	1	3
Output heating oil	3	1	4
Cost	51$	11$	41$

All quantities are in barrels. For example, with the first process, 3 barrels of crude A and 5 barrels of crude B are used to produce 4 barrels of gasoline and 3 barrels of heating oil at a cost of 51 $.

Formulate a linear programming problem that would help the manager maximize net revenue over the next month.

Exercise 2

Solve the following linear program using the simplex method:

$$\begin{aligned} \max \quad & \tfrac{3}{4}x + y \\ \text{subject to} \quad & x + 2y \le 10 \\ & x + y \le 7 \\ & y \le 4 \\ & x, y \ge 0 \end{aligned}$$

Exercise 3

Recall Lemma 3.5. from the lecture: *If the reduced cost vector of a basis B is non-negative and if $x_B^* \ge 0$, then B is an optimal basis.*

Show that the reverse is not true by giving a linear program in equation standard form and an optimal basis with optimal feasible associated basic solution x^* with a negative entry in the reduced cost vector.

Exercise 4

Recall the tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	
$-\frac{3}{4}$	20	$-\frac{1}{2}$	6	0	0	0	3
$\frac{1}{4}$	-8	-1	9	1	0	0	0
$\frac{1}{2}$	-12	$-\frac{1}{2}$	3	0	1	0	0
0	0	1	0	0	0	1	1

from the lecture where the simplex method with the given pivoting rule cycled.

Use the lexicographic pivoting rule to solve the simplex tableau. It is defined as follows:

1. If $\overline{c} \geq 0$, then **output optimal basis** B

2. Otherwise, let j be an index with $\overline{c}(j) < 0$ and let $u = A_B^{-1} A^j$ be the j-th column of the system matrix of the tableau. If $u \leq 0$, then **output** $-\infty$, the linear program is unbounded.

3. Otherwise compute the vectors $[A_B^{-1} b(i) | (A_B^{-1} A)_i] / u(i)$ for all $i = 1, \ldots, m$ with $u(i) > 0$. Let i^* be an index for which this vector is lexicographically smallest.

   Note that $[(A_B^{-1} A)_i | A_B^{-1} b(i)]$ is the ith row in the tableau. For two vectors $u, v \in \mathbb{R}^n$, the vector u is lexicographically smaller than v ($u \leq_{lex} v$), if $u = v$ or if the first nonzero component of $u - v$ is strictly negative.

   The index B_{i^*} leaves the basis and j enters the basis, i.e., the new basis is $B' = B \setminus \{B_{i^*}\} \cup \{j\}$.

4. Update the tableau to

$$\begin{array}{c|c} c^T - c_{B'}^T A_{B'}^{-1} A & -c_{B'}^T x_{B'}^* \\ \hline A_{B'}^{-1} A & x_{B'}^* . \end{array}$$