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Due Date: March 30, 2009
Discussions: March 31, 2009

# Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 6

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**Exercise 1**

A company produces two kinds of products. A product of the first type requires $\frac{1}{4}$ hours of assembly labor, $\frac{1}{8}$ hours of testing, and 1.2 CHF worth of raw materials. A product of the second type requires $\frac{1}{3}$ hours of assembly, $\frac{1}{3}$ hours of testing and 0.9 CHF worth of raw materials. Given the current personnel of the company, there can be at most 90 hours of assembly labor and 80 hours of testing, each day. Products of the first and second type have a market value of 9 CHF and 8 CHF respectively.

1. Formulate a linear programming problem that can be used to maximize the daily profit of the company.

2. Consider the following two modifications to the original problem:

   a) Suppose that up to 50 hours of overtime assembly labor can be scheduled, at a cost of 7 CHF per hour.

   b) Suppose that the raw material supplier provides a 10% discount if the daily bill is above 300 CHF.

   Which of the above two elements can be easily incorporate into the linear programming formulation and how? If one or both are not easy to incorporeate, indicate how you might nevertheless solve the problem.

**Exercise 2**

Solve the following linear program using the simplex method:

$$\begin{aligned} \min \quad & -15x_1 - 60x_2 - 4x_3 - 20x_4 \\ \text{subject to} \quad & 20x_1 + 20x_2 + 10x_3 + 40x_4 \leq 21 \\ & 10x_1 + 30x_2 + 20x_3 \leq 6 \\ & 20x_1 + 40x_2 + 30x_3 + 10x_4 \leq 14 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

*Hint: Introduce slack variables to transform the LP into equality standard form*

**Exercise 3**
Given the following linear program

$$\begin{aligned} \max \quad & 3x_1 + 4x_2 \\ \text{subject to} \quad & -x_1 + x_2 \leq 4 \\ & 2x_1 - x_2 \leq -2 \\ & x_1 - 3x_2 \leq 7 \\ & 4x_1 - 5x_2 \leq 10 \\ & 2x_1 + 6x_2 \leq -12 \\ & x_1, x_2 \leq 0 \end{aligned} \tag{1}$$

compute the value of an optimal solution to (1) following these steps:

1. Formulate the dual linear program.
2. Find an initial solution to the dual LP
3. Solve the dual LP using the simplex method.
4. Obtain the optimal value for the LP (1).
   *Hint: Strong duality*

**Exercise 4**
Consider the following linear program:

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \leq 6 \\ & x_1 + 2x_2 \leq 8 \\ & 3x_1 + 4x_2 \leq 22 \\ & x_1 + 5x_2 \leq 23 \end{aligned}$$

Show that $(4/3, 10/3)$ is an optimal solution by providing a suitable feasible dual solution.