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Discussions: March 31, 2009

Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 7

Exercise 1

Suppose that there are N available currencies, and assume that one unit of currency i can be exchanged for r_{ij} units of currency j. (Naturally we assume that $r_{ij} > 0$.) There are also certain regulations that impose a limit u_i on the total amount of currency i that can be exchanged on any given day (At most u_i units of currency i can be changed to any other currency).

Suppose that we start with B units of currency 1 and that we would like to maximize the number of units of currency N that we end up with at the end of the day, through a sequence of currency transactions. Provide a linear programming formulation of this problem. Assume that for any sequence $i_1, \ldots, i_k$ of currencies, we have $r_{i_1 i_2} \cdot r_{i_2 i_3} \cdots r_{i_{k-1} i_k} \cdot r_{i_k i_1} \leq 1$, which means that wealth cannot be multiplied by going through a cycle of currencies.

Exercise 2

Solve the following linear program using the two-phase simplex method:

$$\begin{aligned} \min \quad & 2x_1 + 3x_2 + 3x_3 + x_4 - 2x_5 \\ \text{subject to} \quad & x_1 + x_2 + 4x_4 + x_5 = 2 \\ & x_1 + 2x_2 + -3x_4 + x_5 = 2 \\ & x_1 - 4x_2 + 3x_3 = 1 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

During the first phase, let the following indices enter the basis in this order: $1, 2, 5$. Use the lexicographic pivoting rule to decide which index will leave the basis in each step.

For each iteration, give the basis and the simplex tableau.

Exercise 3

Solve the following simplex tableau using the dual simplex method.

x_1	x_2	x_3	x_4	x_5	
3	2	1	0	0	0
−2	2	−1	1	0	−3
−2	−1	1	0	1	−1

For each iteration, give the basis and the simplex tableau.

Exercise 4

Consider the following simplex tableau:

x_1	x_2	x_3	x_4	x_5	
δ	-2	0	0	0	-10
-1	η	1	0	0	4
α	-4	0	1	0	1
γ	3	0	0	1	β

The current basic variables are x_3, x_4, x_5. The entries $\alpha, \beta, \gamma, \delta, \eta$ in the tableau are unknown parameters.

For each one of the following statements, find some parameter values that will make the statement true.

1. The current solution is feasible but not optimal.
2. The current solution is optimal.
3. The optimal cost is $-\infty$.

Exercise 5

Consider the following linear programming problem with a single constraint:

$$\begin{aligned} \min \quad & \textstyle\sum_{i=1}^{n} c_i x_i \\ \text{subject to} \quad & \textstyle\sum_{i=1}^{n} a_i x_i = b \\ & x_i \geq 0, \qquad i = 1, \ldots, n. \end{aligned}$$

Here $b, a_i, c_i \in \mathbb{R}$ for each $i = 1, \ldots, n$

1. Derive a simple test for checking the feasibility of this problem.
2. Assuming that the optimal cost is finite, develop a simple method for obtaining an optimal solution directly.