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Due Date: April 28, 2009
Discussions: April 21, 2009

Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 8

Exercise 1
What is the smallest number n such that an algorithm A with running time $1000000 \cdot n^2$ is faster than an algorithm B with a running time of 2^n?

Exercise 2
Decide which ones of the following statements are true and give a short explanation:

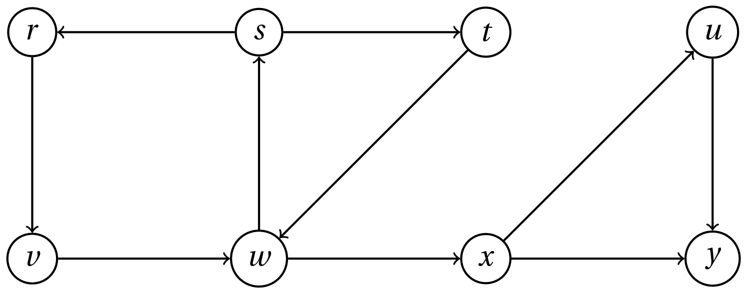
1. $4n^3 + 3n^2 - n - 100 = O\left(n^3\right)$
2. $n = O\left(n^7\right)$
3. $n^3 - 100n^2 = \Omega(n^4)$
4. $2^n = O\left(n^2\right)$
5. $\sqrt{n} = O(n)$
6. $\log(n) = O\left(\sqrt{n}\right)$

Exercise 3
Consider the following algorithm. The input is a sequence of n integers $a[1], \ldots, a[n]$.

```
Require:
for i ← 1 to n do
    for j ← n downto i+1 do
        if a[j] < a[j-1] then
            exchange a[j] ↔ a[j-1]
        end if
    end for
end for
Output a[1],...,a[n].
```

Give an asymptotic upper bound on the running time of the algorithm (with explanations). What is the output of the algorithm?

Exercise 4
Consider the following graph:



Perform the breath first search algorithm on this graph starting in s. For each node v, give the values $\pi[v]$ and $d[v]$ at the end of the algorithm.

Exercise 5

There are two types of professional wrestlers: "good guys" and "bad guys". Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n+r)$-time algorithm that determines whether it is possible to designate some of the wrestlers as good guys and the remainder as bad guys such that each rivalry is between a good guy and a bad guy.

If it is possible to perform such a designation, your algorithm should produce it.

Hint: Describe the rivalries as a graph and use an algorithm from the lecture to solve the problem.