Exercise 1
What is the smallest number $n$ such that an algorithm $A$ with running time $1000000 \cdot n^2$ is faster than an algorithm $B$ with a running time of $2^n$?

Exercise 2
Decide which ones of the following statements are true and give a short explanation:

1. $4n^3 + 3n^2 - n - 100 = O(n^3)$
2. $n = O(n^7)$
3. $n^3 - 100n^2 = \Omega(n^4)$
4. $2^n = O(n^2)$
5. $\sqrt{n} = O(n)$
6. $\log(n) = O(\sqrt{n})$

Exercise 3
Consider the following algorithm. The input is a sequence of $n$ integers $a[1], \ldots, a[n]$.

Require:
1: for $i \leftarrow 1$ to $n$ do
2:    for $j \leftarrow n$ downto $i + 1$ do
3:        if $a[j] < a[j-1]$ then
4:            exchange $a[j] \leftrightarrow a[j-1]$
5:        end if
6:    end for
7: end for
8: Output $a[1], \ldots, a[n]$.

Give an asymptotic upper bound on the running time of the algorithm (with explanations). What is the output of the algorithm?

Exercise 4
Consider the following graph:
Perform the breath first search algorithm on this graph starting in \( s \). For each node \( v \), give the values \( \pi[v] \) and \( d[v] \) at the end of the algorithm.

**Exercise 5**

There are two types of professional wrestlers: "good guys" and "bad guys". Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have \( n \) professional wrestlers and we have a list of \( r \) pairs of wrestlers for which there are rivalries. Give an \( O(n + r) \)-time algorithm that determines whether it is possible to designate some of the wrestlers as good guys and the remainder as bad guys such that each rivalry is between a good guy and a bad guy.

If it is possible to perform such a designation, your algorithm should produce it.

*Hint: Describe the rivalries as a graph and use an algorithm from the lecture to solve the problem.*