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Due Date: May 05, 2009
Discussions: April 28, 2009

Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 9

Exercise 1

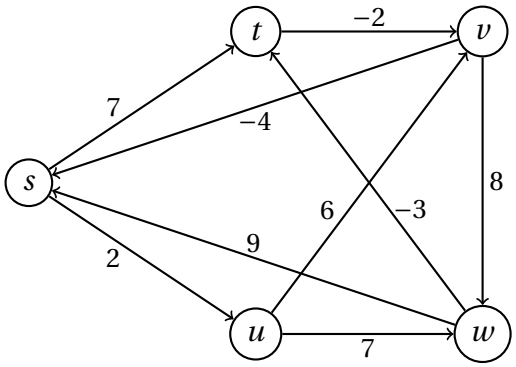
Sort the following functions by their asymptotic growth (i.e. sort them w.r.t. $O(\cdot)$) and give detailed explanations:

$$(3/2)^n \quad n^3 \quad (\log(n))^{\log(n)} \quad 4^{\log(n)}$$

Hint: Use the fact that $x = 2^{\log(x)}$

Exercise 2

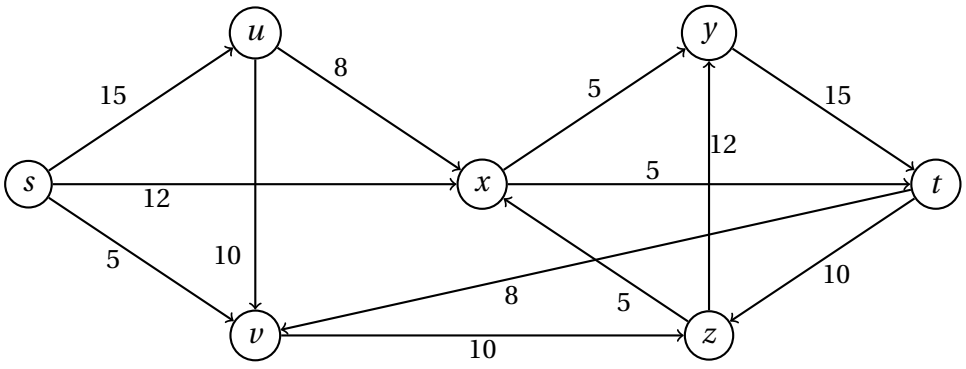
Consider the following graph:



Run the Bellman-Ford algorithm on this graph, using vertex s as the source. For each iteration i give the values $f_i(v)$ and mark the predecessor arcs in the graph.

Exercise 3

Consider the following network:



Run the Ford-Fulkerson algorithm to compute a max $s-t$-flow. For each iteration give the residual network and mark the path you choose for augmentation.

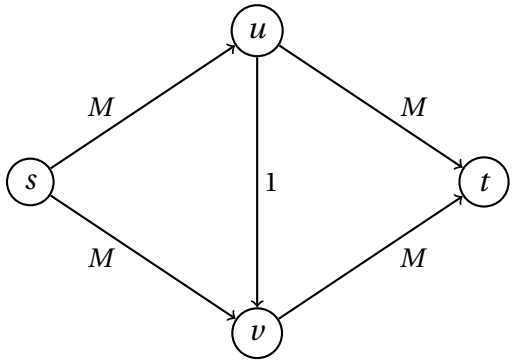
Further give a minimum $s-t$-cut in the network.

Exercise 4

Given a network $D=(V,A)$ with rational capacities $c\colon A\to Q$, show that the Ford-Fulkerson algorithm terminates, i.e. give a bound on its running time (this bound needs not to be polynomial in the input size).

Exercise 5

Consider the following network:



Explain why the Ford-Fulkerson algorithm might take an exponential number of iterations ($2\cdot M$ iterations) if the augmenting paths are chosen in a disadvantageous way.