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Due Date: May 12, 2009
Discussions: May 06, 2009

Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 10

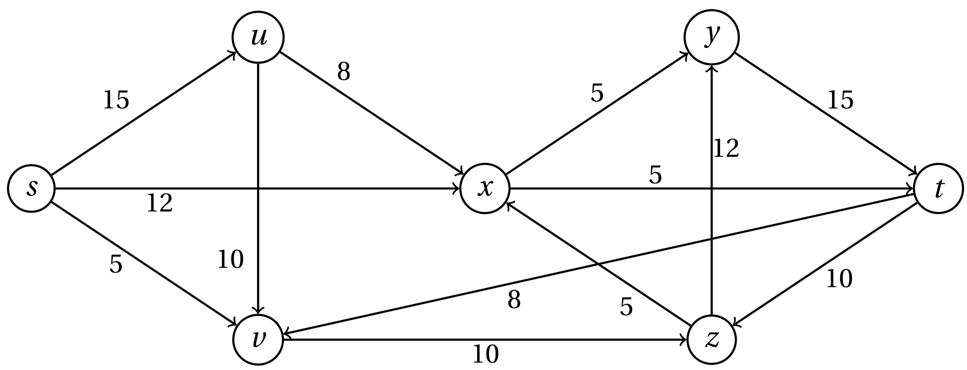
Exercise 1

The Lucky Puck Company has a factory in Vancouver that manufactures hockey pucks, and it has a warehouse in Winnipeg that stocks them. Luck Puck leases space on trucks from another company to ship the pucks from the factory to the warehouse. Because the trucks travel over specified routes between cities and have a limited capacity, Lucky Puck can ship at most $c(u, v)$ crates per day between each pair of cities u and v. Lucky Puck has no control over these routes and capacities and so cannot alter them. Their goal is to determine the largest number p of crates per day that can be shipped from the factory to the warehouse.

Show how to compute p by finding a maximum flow in a network.

Exercise 2

Consider the following network:



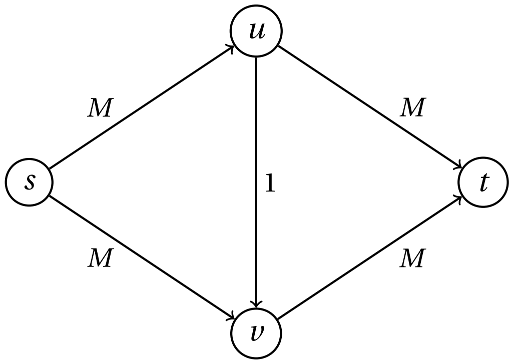
Run the Ford-Fulkerson algorithm to compute a max $s-t$-flow. For each iteration give the residual network and mark the path you choose for augmentation.

Further give a minimum $s-t$-cut in the network.

Exercise 3

Given a network $D = (V, A)$ with rational capacities $c: A \to \mathbb{Q}$, show that the Ford-Fulkerson algorithm terminates even if we do not choose a shortest path for augmentation in each iteration, i.e. give a bound on its running time (this bound needs not to be polynomial in the input size).

Exercise 4
Consider the following network:



Explain why the Ford-Fulkerson algorithm might take an exponential number of iterations ($2 \cdot M$ iterations) if the augmenting paths are chosen in a disadvantageous way.

Exercise 5
An *undirected graph* $G = (V, E)$ is a set of *nodes* together with a set of *edges* $E \subseteq \{\{u, v\} : u, v \in V\}$. G is *connected* if for each pair of nodes $u, v \in V$ there is a path from u to v.

The *edge connectivity* of an undirected graph is the minimum number k of edges that must be removed such that the resulting graph is not connected anymore.

Show how the edge connectivity of an undirected graph $G = (V, E)$ can be determined by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(V)$ vertices and $O(E)$ arcs.

Hint: Consider the bidirected graph $D = (V, A)$, *where* $A = \{(u, v) : \{u, v\} \in E\}$ *(for each edge {u,v} we have the arcs* (u, v) *and* (v, u)*). Is there any relation between the edge connectivity of G and minimum size cuts in D?*