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Due Date: May 26, 2009
Discussions: May 19, 2009

Introduction to Discrete Optimization

Spring 2009

Assignment Sheet 12

Exercise 1

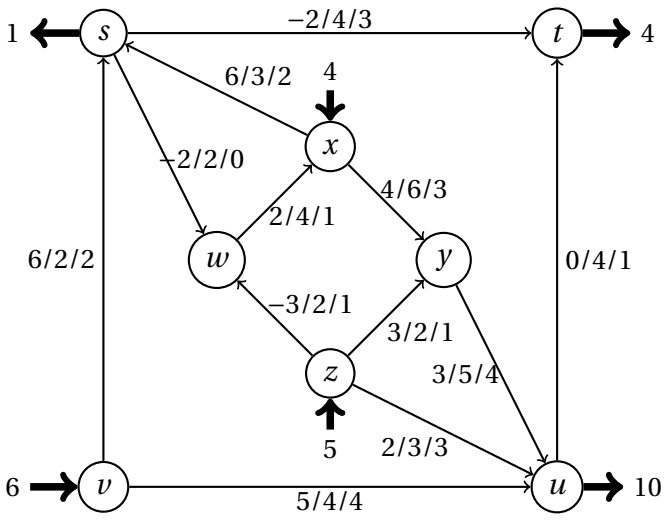
A caterer requires $r_j \in \mathbb{N}$ fresh napkins on each of n successive days, $j = 1, \ldots, n$. He can meet his requirements either by purchasing new napkins or by using napkins previously laundered. Moreover, the laundry has two kinds of service: quick service requires p days and slow service requires q days. Suppose a new napkin costs a cents, quick laundering costs b cents and slow laundering costs c cents. How should the caterer, starting with no napkins, meet his requirements with minimum costs?

Explain how to formulate this as a minimum cost flow problem.

Exercise 2

Consider the following flow network with external flow.

Every arc label is of the form $c(a)/u(a)/x(a)$, where c denote the costs, u the capacities and x a feasible flow.



Draw the corresponding residual network and perform one augmentation of the cycle cancelling algorithm, i.e. choose a negative cycle, augment flow along the cycle and give the resulting residual network.

Exercise 3

Let $D = (V, A)$ be a directed graph with capacities $u\colon A \to \mathbb{Q}_{\geq 0}$, costs $c\colon A \to \mathbb{Q}$ and external flow $b\colon V \to \mathbb{Q}$, and assume that we have an optimal solution $x\colon A \to \mathbb{Q}_{\geq 0}$ for the MCNFP.

Let $a' \in A$, and define a new capacity function

$$u' : A \to \mathbb{Q}_{\geq 0}, a \mapsto \begin{cases} u(a) + 1, & \text{if } a = a' \\ u(a), & \text{else.} \end{cases}$$

Show how to find an optimal solution for the MCNFPs defined by u' efficiently using the solution x.

Hint: Use a shortest path algorithm