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Due Date: May 05, 2009
Discussions: April 28, 2009

Introduction to Discrete Optimization

Spring 2009

Solutions 9

Exercise 1
Sort the following functions by their asymptotic growth (i.e. sort them w.r.t. $O(\cdot)$) and give detailed explanations:

$$(3/2)^n \quad n^3 \quad (\log(n))^{\log(n)} \quad 4^{\log(n)}$$

Hint: Use the fact that $x = 2^{\log(x)}$

Solution
We have

$$\begin{aligned} f(n) &:= (3/2)^n = 2^{\log\left((3/2)^n\right)} = 2^{n\cdot\log(3/2)}, \\ g(n) &:= n^3 = 2^{\log(n^3)} = 2^{3\cdot\log(n)}, \\ h(n) &:= (\log(n))^{\log(n)} = 2^{\log((\log(n))^{\log(n)})} = 2^{\log(n)\cdot\log\log(n)}, \\ i(n) &:= 4^{\log(n)} = 2^{\log(4^{\log(n)})} = 2^{\log(n)\cdot\log(4)}. \end{aligned}$$

Since the function 2^x is monotonically increasing, it is sufficient to sort the exponents by their asymptotic growth.

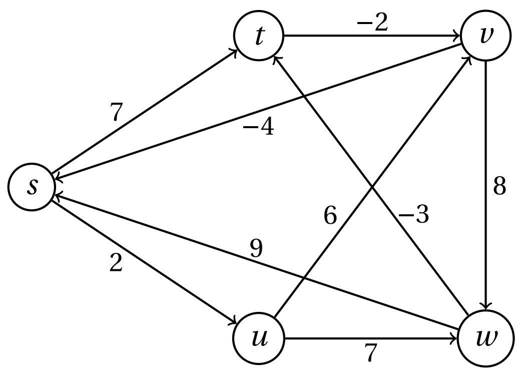
Let $f'(n) := n\cdot\log(3/2)$, $g'(n) := 3\cdot\log(n)$, $h'(n) = \log(n)\cdot\log\log(n)$ and $i'(n) := \log(4)\cdot\log(n)$.

Clearly we have $i'(n) \le g'(n)$ for all $n \ge 0$ since $\log(4) < 3$. Further $g'(n) \le h'(n)$ for all $n \ge 256$ since $\log\log(n) \ge 3$ for all $n \ge 256$. Finally for n large enough we have

$$h'(n) = \log(n)\cdot\log\log(n) \le \log(n)\cdot\log(n) = (\log(n))^2 \le \log(3/2)(\sqrt{n})^2 = \log(3/2)\,n = f'(n).$$

Thus $4^{\log(n)} = O(n^3)$, $n^3 = O\left((\log(n))^{\log(n)}\right)$, $(\log(n))^{\log(n)} = O\left((3/2)^n\right)$

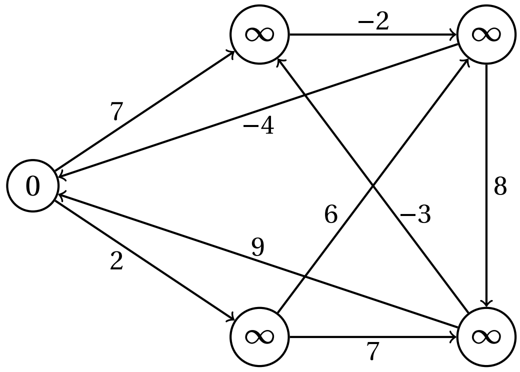
Exercise 2
Consider the following graph:



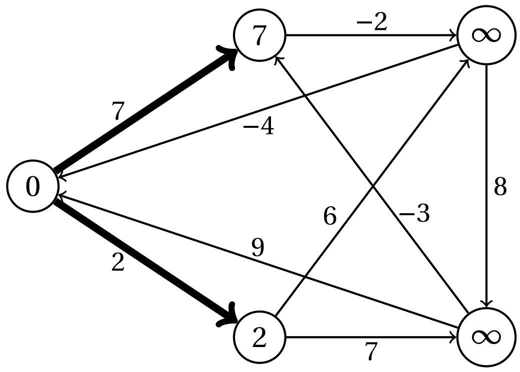
Run the Bellman-Ford algorithm on this graph, using vertex s as the source. For each iteration i give the values $f_i(v)$ and mark the predecessor arcs in the graph.

Solution

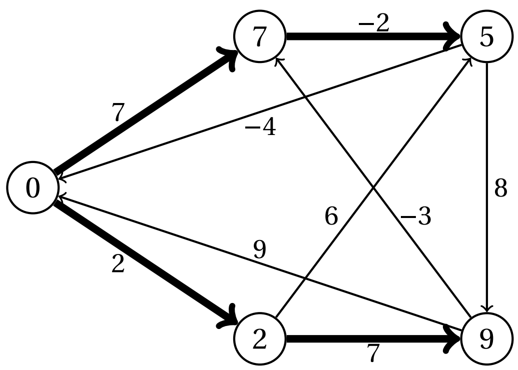
The $f_i(v)$ values are the node label, and predecessors are marked as thick arcs. Initialization:



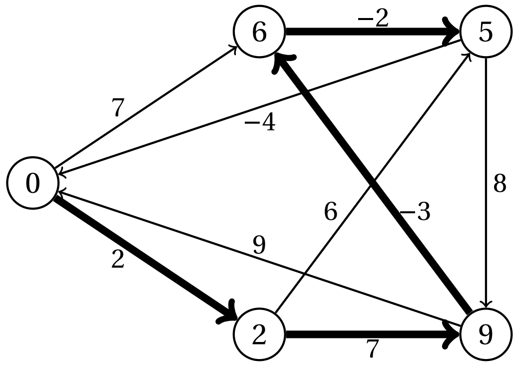
After 1. iteration:



After 2. iteration:



After 3. iteration:



After 4. iteration: