ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Institut d'Analyse et Calcul Scientifique (IACS)
Section Mathématiques

SEMINAIRE D'ANALYSE

- ***VENDREDI 27 mars 2009 à 16h00 à la salle MA A330***

Monsieur ***Scipio CUCCAGNA*** *(Université de Modena, Italie)* donnera une conférence sur le thème:

"DISPERSION OF SMALL ENERGY SOLUTIONS OF THE NONLINEAR KLEIN GORDON EQUATION"

In this talk I discuss a joint paper with Dario Bambusi in which we expand significantly and we simplify a paper by A.Soffer and M.Weinstein [SW]. We consider a nonlinear Hamiltonian Klein Gordon Equation (NLKG) where the linear part involves not the Laplacian, but rather a Schroedinger operator H. We then show that if H is generic in some sense, any solution of the NLKG which initially has small energy, eventually disperses and looks like a solution of the linear KG equation. Such dispersion and scattering results can be simple consequences of the theory of Strichartz inequalities if H does not have eigenvalues. In this case dispersion and scattering are obtained viewing the NLKG as a perturbation of the linear KG. But if H has eigenvalues, then there could be small discrete components fluctuating and not decaying. It turns out that, by nonlinear coupling, energy leaks from the discrete modes in the continuous modes were it scatters. The exact mechanism is related to the simple fact that sufficiently large multiples of the eigenvalues of the linear KG are contained in the continuous spectrum. This yields some dissipation in the equations of the discrete modes, through a mechanism reminiscent of the classical Fermi Golden Rule. A first appearance of this idea is in [S] and in [BS]. [SW] discusses the case when H has just one eigenvalue, with some strong restriction on its position. The derivation of the Fermi Golden Rule in [SW] and the closing up of the nonlinear estimates is quite complicated. We simplify substantially the argument, we relax most spectral hypotheses on H allowing any number of possibly multiple eigenvalues with only mild restrictions. We use crucially the Hamiltonian structure of the NLKG.

[BS] V.Buslaev, G.Perelman, On the stability of solitary waves for nonlinear Schroedinger equations, Nonlinear evolution equations, editor N.N. Uraltseva, Transl. Ser. 2, 164, Amer. Math.
Soc., pp. 75--98, Amer. Math. Soc., Providence (1995).
[S] I.M.Sigal, Nonlinear wave and Schrödinger equations. I. Instability of periodic and quasi-periodic solutions, C.M.P., vol 153 (1993), 297-320
[SW] A.Soffer, M.I.Weinstein, Resonances, radiation damping and instability in Hamiltonian nonlinear wave equations, Invent. Math., 136 (1999), pp. 9--74.

Lausanne, le 23 janvier 2009
BD/BB/VL

_____
Les séminaires qui ont lieu à la Section de Mathématiques sont annoncés sur Internet à l'adresse WWW (http:// www.epfl.ch/cgi-bin/memento/memento)