SEMINAIRE D'ANALYSE

➢ *JEUDI 12 JANVIER 2017 à 14h15 - salle MA A112*

Professeur **GYULA CSATO** (Universidad de Concepción Matemática, Chili) donnera une conférence sur le thème:

« About Hardy-Sobolev, Moser-Trudinger and isoperimetric inequalities with weights»

Abstract

The well known Sobolev embedding states that if $1 < p < n$, then there exists a constant $C > 0$ such that

$$\|u\|_{L^{p^*}(\mathbb{R}^n)} \le C\|Du\|_{L^p(\mathbb{R}^n)} \quad \forall u \in C_c^\infty(\mathbb{R}^n), \qquad p^* = \frac{np}{n-p}.$$

In the limiting case $p = n$, the corresponding result in a bounded domain Ω is the Moser-Trudinger embedding and it can be stated as follows (here for simplicity in dimension 2):

$$\sup_{u \in C_c^\infty(\Omega),\, \|Du\|_{L^2} \le 1} \int_\Omega e^{\alpha|u|^2} \le C(\Omega), \quad 0 \le \alpha \le 4\pi$$

for some constant $C = C(\Omega)$. These embeddings have many weighted versions, for instance one of them is, if $1 < p < n$,

$$\left\|\frac{u}{|x|^\tau}\right\|_{L^{p^*}(\mathbb{R}^n)} \le C\|Du\|_{L^p(\mathbb{R}^n)} \quad \forall u \in C_c^\infty(\mathbb{R}^n), \qquad 0 \le \tau \le 1, \quad p^* = \frac{np}{n-p+p\tau}.$$

There are many more types of weighted versions and in general these are called Hardy-Sobolev or Caffarelli-Kohn-Nirenberg inequalities. There is also a corresponding weighted version for the Moser-Trudinger embedding, called the singular Moser-Trudinger embedding. First I will give a short overview on these embeddings, on best constants and extremal functions. I will mention briefly some of their geometric versions, called isoperimetric inequalities with densities, respectively some recent results on the subject. Then I will mainly concentrate on the Moser-Trudinger and singular Moser-Trudinger embeddings. Towards the end of my talk I will present some of my own results appearing in the following references:

References

[1] CSATÓ G., *An isoperimetric problem with density and the Hardy-Sobolev inequality in* $\mathbb{R}^2$, Differential Integral Equations, **28**, Number 9/10 (2015), 971–988.

[2] CSATÓ G. AND ROY P., *Extremal functions for the singular Moser-Trudinger inequality in 2 dimensions*, Calc. Var. Partial Differential Equations, **54**, Issue 2 (2015), 2341–2366.

[3] CSATÓ G. AND ROY P., *The singular Moser-Trudinger inequality on simply connected domains*, Comm. Partial Differential Equations, to appear.

Lausanne, le 22 décembre 2016
BD/vl