ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

B. Buffoni – B. Dacorogna – J. Krieger - H.M Nguyên - Section Mathématiques

SEMINAIRE D'ANALYSE

➢ ***VENDREDI 27 OCTOBRE 2017 à* 14h15 - *salle MA A3 31***

Professeur ***BRUNO FRANCHI*** *(*Université de Bologne, Italie) donnera une conférence sur le thème:

«DIFFERENTIAL FORMS IN HEISENBERG GROUPS AND SOBOLEV-POINCARE INEQUALITIES»

In collaboration with A. Baldi, P. Pansu and with R. Serapini

The Heisenberg group $\mathbb{H}^n$ is the $(2n+1)$-dimensional Lie group with nilpotent, stratified Lie algebra $\mathfrak{h}$ of step 2 given by

$$\mathfrak{h} = \text{span}\,\{X_1, \dots, X_n, Y_1, \dots, Y_n\} \oplus \text{span}\,\{T\} := \mathfrak{h}_1 \oplus \mathfrak{h}_2,$$

where the only nontrivial commutation rules are $[X_j, Y_j] = T$ for any $j = 1, \dots, n$.

It is well-known that $\mathbb{H}^n$ can be identified with $\mathbb{R}^{2n+1}$ through the exponential map. The stratification of the algebra induces a family of nonisotropic dilations δ_λ in the group, again via the exponential map. However, differential forms on $\mathfrak{h}$ split into 2 eigenspaces under δ_λ, so that de Rham complex lacks scale invariance under these anisotropic dilations.

A substitute for de Rham's complex, that recovers scale invariance under δ_λ has been defined by M. Rumin 1994. In this talk we shall give a gist of his original construction, as well as its interpretation in terms of "linear manifolds" in Heisenberg groups in the setting of Geometric Measure Theory in stratified nilpotent Lie groups. Albeit homotopic to the de Rham complex, this complex shows exterior differentials which are differential operators of higher order.

Finally, we shall prove Sobolev and Poincaré inequalities within Rumin's complex: they provide a natural extension of the corresponding classical inequalities for functions in Euclidean spaces and are a quantitative formulation of the fact that closed forms are locally exact.

Lausanne, le 3 octobre 2017
BD/vl

Les séminaires d'Analyse sont annoncés sur le site caa.epfl.ch et sur http://memento.epfl.ch/maths/