Lecture 1. Dror-Farjoun localization

A. Geometric localization

M = pointed space

$X, Y, \ldots$, $A, B, \ldots$ = simply connected pted spaces

★ a) X is $* \to M$ local (M-null):

$$\mathrm{map}(M, X) \xrightarrow{\simeq_{wk}} \mathrm{map}(*, X) = X \text{ weak eq}$$

$$\iff \mathrm{map}_*(M, X) \simeq_{wk} *$$

$$\iff \forall i \geq 0,\ 0 = \pi_i \mathrm{map}_*(M, X) \cong [\Sigma^i M, X]_*$$

Note: fibration $\mathrm{map}_a(M, X) \to \mathrm{map}(M, X) \xrightarrow{ev} X$

★ b) $A \to B$ is a local equivalence:

$$\mathrm{map}_*(A, X) \xleftarrow{\simeq_{wk}} \mathrm{map}_*(B, X) \quad \forall \text{ local } X$$

$$\iff \mathrm{map}(A, X) \xleftarrow{\simeq_{wk}} \mathrm{map}(B, X) \quad \forall \text{ local } X$$

★ e.g. $* \to M$ is a local equivalence