b) $A \to B$ is an M-local equivalence:

$\forall$ local Y, $map_*(A,Y) \xleftarrow{\simeq\, whe} map_*(B,Y)$

$\Leftrightarrow$ $\forall i \geq 0$, $[\Sigma^i A, Y] \xleftarrow{\simeq} [\Sigma^i B, Y]_*$, $\forall$ loc Y

(WLOG: $A \subseteq B$) $\Leftrightarrow$

obstructions vanish $\forall$ local Y:

$H^*(\Sigma^i B, \Sigma^i A; \pi_* Y) = 0$ $\Leftrightarrow$

$H^*(B, A; D) = 0$ $\forall$ local $D \cong D \otimes R$

$\Leftrightarrow$ (univ coeff thm) $H_*(B,A) \otimes R = 0$

$\Leftrightarrow$ $H_*(A) \otimes R \xrightarrow{\simeq} H_*(B) \otimes R$

Summary: $M = M(\bigoplus_{q \in \mathcal{J}} \mathbb{Z}/q)$, $R = \mathbb{Z}[\mathcal{J}^{-1}]$.

1) X is M-local: $\pi_* X$ is local $\Leftrightarrow$ $\pi_* X \cong \pi_* X \otimes R$

2) $A \to B$ is a local equivalence: $H_* A \otimes R \cong H_* B \otimes R$

exercise: 3) $X \to LX = X \otimes R$ is localization: $\pi_*(X \otimes R) = \pi_* X \otimes R$

$H_*(X \otimes R) = H_* X \otimes R$