c) $X \xrightarrow{i} LX = L_M X$ is $\underline{M}$-localization:

1) LX is local
2) i is a local equivalence

Theorem: $X \xrightarrow{i} LX$ exists, is unique up to homotopy, and is a functor on the homotopy category.

Note: In fact, there is a strict functor for LX.

B. Geometric localization exists

M = ptd space, X = s.c. ptd space

Will assume M is a countable CW complex

Consider the Barratt-Puppe sequence:

$$\bigvee_{\substack{\text{all } i\geq 0 \\ \text{all } f:\, \Sigma^i M \to X}} \Sigma^i M \xrightarrow{F} X \to C_F \longrightarrow \bigvee \Sigma^{i+1} M$$

$$M_\sigma \xrightarrow{F} X \to X_* \longrightarrow \Sigma M_\sigma$$