Barratt-Puppe sequence: "coexact"

$$\Sigma^i M_\sigma \to \Sigma^i X \to \Sigma^i X_+ \to \Sigma^{i+1} M_\sigma$$

$\forall$ local Y, $i \geq 0$: $\quad [\Sigma^i M_\sigma, Y]_* = 0$

exact sequence $\Rightarrow$:

$$0 \leftarrow [\Sigma^i X, Y]_* \xleftarrow{\cong} [\Sigma^i X_+, Y] \leftarrow 0$$

☆ $\therefore$ $X \to X_+$ is a local equiv and:

all $\Sigma^i M \to X \to X_+$ are $\simeq *$

Construction of LX by transfinite recursion:

α = ordinal

$X_0 = X$

$X_{\alpha+1} = (X_\alpha)_+$: $\alpha+1$ = successor ordinal

$X_\beta = \varinjlim_{\alpha<\beta} X_\alpha = \bigcup_{\alpha<\beta} X_\alpha$: β = limit ordinal.

$LX = X_\Omega$: Ω = first uncountable ordinal

☆ i.e. $X = X_0 \to X_1 \to X_2 \to \cdots \to X_{\omega_0} \to X_{\omega_0+1} \to \cdots$

$\to X_\cdot \to \cdots \to \lim_\alpha X_\alpha = X_\Omega$