Claim: $X \xrightarrow{i} LX$ is localization

1) i is a local equivalence: $\forall$ local Y

$$
\begin{aligned}
map_*(X_\Omega, Y) &= map_*(\varinjlim X_\alpha, Y) \\
&= \varprojlim map_*(X_\alpha, Y) \\
&\cong \varprojlim map_*(X, Y) \\
&\cong map_*(X, Y)
\end{aligned}
$$

2) LX is local:

M countable CW $\Rightarrow$ $M = \bigcup M_n$, M_n finite $\subseteq M_{n+1}$

let $\Sigma^i M \to X_\Omega$:

$\forall n$, $\exists \alpha(n)$ s.t. $\Sigma^i M_n \to X_{\alpha(n)}$

$$
\begin{aligned}
\therefore \quad \Sigma^i M \to \bigcup X_{\alpha(n)} &= \varinjlim X_{\alpha(n)} \\
&= X_{\underline{\lim}\, \alpha(n)} = X_\beta
\end{aligned}
$$

$\beta < \Omega$ since Ω is not a countable limit of lesser ordinals