

### C. Localization of abelian groups

$M =$  abelian group

$X, Y, \dots, A, B, \dots =$  abelian groups

★ a)  $X$  is  $0-M$  local (or  $M$ -null):

$$0 = \text{Ext}^*(M, X) = \begin{cases} \text{hom}(M, X) & * = 0 \\ \text{Ext}(M, X) & * = 1 \end{cases}$$

★ b)  $A \rightarrow B$  is a local equivalence:

$$\forall \text{ local } X, \text{hom}(A, X) \xrightarrow{\cong} \text{hom}(B, X)$$

(strong local equivalence:  $\forall \text{ local } X$

$$\text{Ext}^*(A, X) \xrightarrow{\cong} \text{Ext}^*(B, X)$$

★ c)  $X \xrightarrow{i} LX = L_M X$  is  $0-M$  localization.

1)  $LX$  is local

2)  $i$  is a local equivalence