D. Classical localization of abelian groups
= inverting a set of primes

Let $\mathcal{S}$ = a set of primes $\subseteq \mathbb{Z}$

$$M_{\mathcal{S}} = \bigoplus_{q \in \mathcal{S}} \mathbb{Z}/q$$

a) X is $M_{\mathcal{S}}$-local:

$$0 = \mathrm{Ext}^{*}(M_{\mathcal{S}}, X) = \begin{cases} \hom(M_{\mathcal{S}}, X) = \prod_{q \in \mathcal{S}} \ker q : X \to X \\ \mathrm{Ext}(M_{\mathcal{S}}, X) = \prod_{q \in \mathcal{S}} X/qX \end{cases}$$

$\iff \forall q \in \mathcal{S}$: X is uniquely q-divisible

$\iff$ X is a module over $\mathbb{Z}[\mathcal{S}^{-1}]$

$\iff$ $X \cong X \otimes \mathbb{Z}[\mathcal{S}^{-1}]$

e.g. $\mathcal{S} = \{p\}$, $\mathbb{Z}[\mathcal{S}^{-1}] = \mathbb{Z}[\frac{1}{p}]$,
local away from p

e.g. $\mathcal{S}$ = all primes $\mathbb{P}$, $\mathbb{Z}[\mathcal{S}^{-1}] = \mathbb{Q}$,
rational

e.g. $\mathcal{S} = \mathbb{P} - \{p\}$, $\mathbb{Z}[\mathcal{S}^{-1}] = \mathbb{Z}_{(p)}$,
local at p