

★ b)  $A \rightarrow B$  is a local equivalence:

$$\forall \text{ local } X = X \otimes \mathbb{Z}[\mathcal{S}^{-1}]$$

$$\text{hom}(A, X) \xrightarrow{\cong} \text{hom}(B, X) \iff$$

$$\text{hom}(A \otimes \mathbb{Z}[\mathcal{S}^{-1}], X) \xrightarrow{\cong} \text{hom}(B \otimes \mathbb{Z}[\mathcal{S}^{-1}], X)$$

$$\iff A \otimes \mathbb{Z}[\mathcal{S}^{-1}] \xrightarrow{\cong} B \otimes \mathbb{Z}[\mathcal{S}^{-1}]$$

★ c) localization exists:

$$X \rightarrow LX = X \otimes \mathbb{Z}[\mathcal{S}^{-1}]$$

1)  $LX$  is local

2)  $\sigma$  is a local equiv.

e.g.  $\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Z}_{(p)} = \mathbb{Z}_{(p)}$

$$\mathbb{Z}/p^r \otimes_{\mathbb{Z}} \mathbb{Z}_{(p)} = \mathbb{Z}/p^r$$

$$\mathbb{Z}/p \otimes_{\mathbb{Z}} \mathbb{Z}_{(p)} = 0, (\mathbb{Z}/p)_{(p)} = 1$$