E. Classical localization of spaces

$\mathcal{J}$ = set of primes to be inverted, $R = \mathbb{Z}[\mathcal{J}^{-1}]$, $G \cdot M_{\mathcal{J}} = \bigoplus_{q \in \mathcal{J}} \mathbb{Z}/q$

X = s.c. gtd space

$M = M(G,1)$ = Moore space: $H_1(M(G,1)) \approx G$, $\tilde{H}_i(M(G,1)) = 0$, $i \neq 1$

e.g. $M(\mathbb{Z}/2,1) = \mathbb{R}P^2 = S^1 \cup_2 e^2$

$$M(G,1) = \bigvee_{\alpha} S^1_\alpha \cup \bigcup_{\beta} e^2_\beta$$

α gen $\qquad$ β relation.

$$M_{\mathcal{J}} = \bigvee_{q \in \mathcal{J}} M(\mathbb{Z}/q,1)$$

★ a) X is M-local $\Leftrightarrow$ X is $M(\mathbb{Z}/q,1)$-local $\forall q \in \mathcal{J}$

Barratt-Puppe: $S^1 \xrightarrow{q} S^1 \to M(\mathbb{Z}/q,1) \to S^2 \xrightarrow{q} S^2$

$$S^{1+i} \xrightarrow{q} S^{1+i} \to \Sigma^i M(\mathbb{Z}/q,1) \hookrightarrow S^{2+i} \xrightarrow{q} S^{2+i}$$