$\therefore$ exact

$$\pi_{1+i}(X) \xleftarrow{q} \pi_{1+i}(X) \leftarrow [\Sigma^i M(\mathbb{Z}/q,1), X]_* \leftarrow \pi_{2+i} X \xleftarrow{q} \pi_{2+i} X$$

$$0 \leftarrow \ker\{\pi_{1+i}(X) \circlearrowleft q\} \leftarrow [\Sigma^i M(\mathbb{Z}/q,1), X]_*$$

$$\hookleftarrow \pi_{2+i} X / q\pi_{2+i} X \leftarrow 0$$

$$0 \leftarrow \hom(\mathbb{Z}/q, \pi_{1+i}(X)) \leftarrow [\Sigma^i M(\mathbb{Z}/q,1), X]_*$$

$$\hookleftarrow \mathrm{Ext}(\mathbb{Z}/q, \pi_{2+i} X) \leftarrow$$

in general:

$$0 \leftarrow \hom(G, \pi_{1+i} X) \leftarrow [\Sigma^i M(G,1), X]_*$$

$$\hookleftarrow \mathrm{Ext}(G, \pi_{i+2} X) \leftarrow 0$$

$\therefore$ X is local $\iff [\Sigma^i M(\mathbb{Z}/q,1), X]_* = 0 \ \forall i$

$\iff \pi_i X$ is uniquely q-divisible $\forall q \in \mathcal{S}$

$\iff \pi_* X \cong \pi_* X \otimes R$