

Lecture 2. p -completion

$X, Y, \dots, A, B, \dots =$ abelian groups, $p =$ prime

3 questions:

a) What does it mean to be p -complete?

b) What does it mean for $A \rightarrow B$ to be a p -complete equivalence?

c) What is p -completion?

Answer when A is finitely generated,

$$\hat{A}_p = \varprojlim A/p^n A$$

$$\mathbb{Z}[\frac{1}{p}] = \cup \left\{ \frac{x}{p^n} : n \geq 0, x \in \mathbb{Z} \right\}$$

$$= \{x\} \cup \left\{ \frac{x}{p} \right\} \cup \left\{ \frac{x}{p^2} \right\} \cup \dots$$

$$= \varinjlim \mathbb{Z} \xrightarrow{p} \mathbb{Z} \xrightarrow{p} \mathbb{Z} \xrightarrow{p} \dots$$

$$\mathbb{Z}(p^\infty) = \frac{\mathbb{Z}[\frac{1}{p}]}{\mathbb{Z}} = \varinjlim 0 \rightarrow \mathbb{Z}/p \xrightarrow{p} \mathbb{Z}/p^2 \xrightarrow{p} \dots$$

$=$ p -primary component (\mathbb{Q}/\mathbb{Z})