☆ a) X is p-complete:

$$0 = \mathrm{Ext}^*\left(\mathbb{Z}[\tfrac{1}{p}], X\right) = \begin{cases} \hom(\mathbb{Z}[\tfrac{1}{p}], X), & * = 0 \\ \mathrm{Ext}(\mathbb{Z}[\tfrac{1}{p}], X), & * = 1 \end{cases}$$

$$= \begin{cases} \hom(\varinjlim \mathbb{Z}, X) \\ \mathrm{Ext}(\varinjlim \mathbb{Z}, X) \end{cases} = \begin{cases} \varprojlim \hom(\mathbb{Z}, X) \\ \varprojlim^1 \hom(\mathbb{Z}, X) \end{cases}$$

$$= \begin{matrix} \varprojlim \\ \varprojlim^1 \end{matrix} \left(X \xleftarrow{p} X \xleftarrow{p} X \xleftarrow{p} X \cdots \right)$$

Note: ~~exact~~

$$0 \to \varprojlim^1 \hom(A_n, X) \to \mathrm{Ext}(\varinjlim A_n, X) \to \varprojlim \mathrm{Ext}(A_n, X) \to 0$$

e.g. $(\ell, p) = 1 \Rightarrow \mathbb{Z}/\ell$ not p-complete, $\varprojlim \neq 0$

$X = \mathbb{Z}/p^n\mathbb{Z}$ is p-complete since: $\exists n$ s.t. $p^n \equiv 0$ on X

$\Rightarrow \varprojlim = 0$

$\varprojlim^1 = 0$ by Mittag-Leffler

$\mathbb{Z}$ is not p-complete: $\varprojlim = 0$, $\varprojlim^1 \neq 0$