Mittag-Leffler $\Rightarrow \varprojlim^1 = 0$:

$\cdots A_{n+1} \xrightarrow{f_{n+1}} A_n \to \cdots$: $\forall n$, image $(f_{n+k} \cdots f_{n+1})$

$\subseteq A_n$ is eventually constant

b) $f : A \to B$ is a p-complete equivalence:

$\forall$ p-complete X, $\hom(A, X) \xleftarrow{\cong} \hom(B, X)$

Note: strong p-complete equiv

$\forall$ p-complete X,

$$\mathrm{Ext}^*(A, X) \xleftarrow{\cong} \mathrm{Ext}^*(B, X)$$

Exercise: $\ker f$, $\operatorname{coker} f$ $\mathbb{Z}[\frac{1}{p}]$ modules

$\Rightarrow f$ strong p-complete equivalence.

c) What is p-completion of X? Answer: δ below

exact: $0 \to \mathbb{Z} \to \mathbb{Z}[\frac{1}{p}] \to \mathbb{Z}(p^\infty) \to 0 \Rightarrow$

exact:

$$0 \to \hom(\mathbb{Z}(p^\infty), X) \to \hom(\mathbb{Z}[\tfrac{1}{p}], X) \to \hom(\mathbb{Z}, X) = X$$

$$\xrightarrow{\delta} \mathrm{Ext}(\mathbb{Z}(p^\infty), X) \to \mathrm{Ext}(\mathbb{Z}[\tfrac{1}{p}], X) \to 0$$

$$\mathrm{Ext}(\mathbb{Z}(p^\infty), X) = \hat{X}_p$$