☆ 1) $\hat{X}_p = \mathrm{Ext}(\mathbb{Z}/p^\infty, X)$ is p-complete:

Cartan-Eilenberg: A, B, C abelian groups
$C \to Q_*$ injective resol
$P_* \to A$ projective resol

$$Y = \hom(P_* \otimes B, Q_*) = \hom(P_*, \hom(B, Q_*))$$

two spectral sequences $\Rightarrow H(Y)$:

$$\underline{E}^2 = \mathrm{Ext}^*(\mathrm{Tor}_*(A, B), C) = \underline{E}^\infty$$

$$E^2 = \mathrm{Ext}^*(A, \mathrm{Ext}^*(B, C)) = E^\infty$$

$$E^2 = 0 \Leftarrow \underline{E}^2 = 0$$

$$\therefore \ \mathrm{Tor}_*(\mathbb{Z}[\tfrac{1}{p}], \mathbb{Z}/p^\infty) = 0 \Rightarrow$$

$$\mathrm{Ext}^*(\mathbb{Z}[\tfrac{1}{p}], \mathrm{Ext}^*(\mathbb{Z}/p^\infty, X)) = 0$$

☆ 2) $X \xrightarrow{\delta} \hat{X}_p = \mathrm{Ext}(\mathbb{Z}/p^\infty, X)$ is a p-complete equiv

factor exact:

$$\hom(\mathbb{Z}[\tfrac{1}{p}], X) \to X \to \mathrm{Ext}(\mathbb{Z}/p^\infty, X) \to \mathrm{Ext}(\mathbb{Z}[\tfrac{1}{p}], X) \to 0$$

$$\begin{tikzcd} X \arrow[dr, "k"] & & \mathrm{Ext}(\mathbb{Z}/p^\infty, X) \\ & D \arrow[ur, "j"] \arrow[dr] & \\ 0 \arrow[ur] & & 0 \end{tikzcd}$$