mult by p isom on $\mathbb{Z}[\frac{1}{p}]$ $\Rightarrow$

$\mathrm{Ext}^*(\mathbb{Z}[\frac{1}{p}], X)$ is a $\mathbb{Z}[\frac{1}{p}]$-module

$\Rightarrow$ $\forall$ p-complete Y, $\mathrm{Ext}^*(\mathrm{Ext}^*(\mathbb{Z}[\frac{1}{p}], X), Y) = 0$

$\Rightarrow$ j strong p-complete equiv, k (hom) p-complete equiv

☆ More about $\hat{X}_p = \mathrm{Ext}(\mathbb{Z}/p^\infty, X)$:

$$\mathbb{Z}(p^\infty) = \varinjlim \ \mathbb{Z}/p \xrightarrow{p} \mathbb{Z}/p^2 \xrightarrow{p} \mathbb{Z}/p^3 \xrightarrow{p} \cdots$$

exact:

$$\begin{tikzcd}
0 \arrow[r] & \lim^1 \hom(\mathbb{Z}/p^r, X) \arrow[r] \arrow[d, equal] & \mathrm{Ext}(\mathbb{Z}/p^\infty, X) \arrow[r] \arrow[d, equal] & \varprojlim \mathrm{Ext}(\mathbb{Z}/p^r, X) \arrow[r] \arrow[d, equal] & 0 \\
& \varprojlim^1 \left(X_p \xleftarrow{p} X_{p^2} \xleftarrow{p} X_{p^3} \leftarrow \cdots \right) & \hat{X}_p & \varprojlim X/p^rX &
\end{tikzcd}$$

$= 0$ ~~whenever~~: X ~~...~~ finitely gen $\Rightarrow$ p-torsion has bounded order $\Rightarrow$

$\varprojlim^1 = 0$

☆ $\therefore$ X fin gen $\Rightarrow$ $\hat{X}_p \cong \varprojlim X/p^rX$