e.g. $S^n \cup_{p^r} e^{n+1}$ is p-complete: all π_* are p-torsion

$S^n \cup_\ell e^{n+1}$ not p-complete, $(\ell, p) = 1$

S^n not p-complete

2) $A \xrightarrow{f} B$ is a p-complete equiv:

$\mathrm{map}_*(A, X) \xleftarrow{\simeq\, wk} \mathrm{map}_*(B, X)$ $\forall$ p-complete X

$\Leftrightarrow$ $\forall i \geq 0$, p-complete X: $[\Sigma^i A, X]_* \xleftarrow{\cong} [\Sigma^i B, X]_*$

$\Leftrightarrow$ (WLOG: $A \subseteq B$)

$H^*(B, A; D) = 0$ $\forall$ p-complete D

$\Leftrightarrow$ $\mathrm{Ext}^*(H_*(B, A), D) = 0$ $\forall$ p-complete D.

$\Leftrightarrow$ (Exercise below) $H_*(B, A)$ is a $\mathbb{Z}[\frac{1}{p}]$ module

$\Leftrightarrow$ $H_*(B, A)$ is uniquely p-divisible

$\Leftrightarrow$ $H_*(B, A; \mathbb{Z}/p) = 0$ (UCT:
$0 \to H_* \otimes \mathbb{Z}/p$
$\downarrow$
$H_*(\ ; \mathbb{Z}/p)$
$\downarrow$
$\mathrm{Tor}(H_{*-1}, \mathbb{Z}/p)$
$\downarrow$
0.)

$\Leftrightarrow$ $H_*(A; \mathbb{Z}/p) \xrightarrow{\cong} H_*(B; \mathbb{Z}/p)$

$\Leftrightarrow$ $A \to B$ is a mod p-homology isom.