Note: $\pi_* X$ finite gen $\Longrightarrow$

$$\pi_*(\hat{X}_p) = (\pi_* X)\hat{}_p$$

(since true for Eilenberg-Maclane spaces)

But: exact $0 \to \mathbb{Z} \to \mathbb{Z}[\frac{1}{p}] \to \mathbb{Z}/p^\infty \to 0$

gives fibration sequence

$$K(\mathbb{Z}[\tfrac{1}{p}],1) \to K(\mathbb{Z}/p^\infty,1) \to K(\mathbb{Z},2) \to K(\mathbb{Z}[\tfrac{1}{p}],2)$$

1) $K(\mathbb{Z}[\frac{1}{p}],n)$ has 0 mod p homology

2) $\therefore$ $K(\mathbb{Z}/p^\infty,1) \to K(\mathbb{Z},2)$

↑ mod p homology isom

3) $K(\mathbb{Z},2) \to K(\hat{\mathbb{Z}}_p,2)$

↑ mod p homotopy isom

$\therefore$ mod p homology isom

4) $\therefore$ $K(\mathbb{Z}/p^\infty,1) \to K(\mathbb{Z},2) \to K(\hat{\mathbb{Z}}_p,2)$

$$K(\hat{\mathbb{Z}}_p,2) = K(\mathbb{Z}/p^\infty,1)\hat{}_p$$