Localization

Theorem. Y simply connected $+$ $H_*(Y, \mathbb{Z}/p) = 0$ $\forall * \gg 0$
$+$ $\pi_2 Y$ torsion $\Longrightarrow$

$$LY\langle n\rangle = \hat{Y}_p \quad \forall n$$

where $L = L_{B\mathbb{Z}/p \,\vee\, M(\mathbb{Z}[\frac{1}{p}],1)}$

Corollary (Serre's theorem) : Y = simply connected finite complex, $\pi_i Y = 0$ $\forall i \gg 0$
$\Longrightarrow$ Y contractible.

Proof:

1) Assume $\pi_2 Y$ is finite and let p be a prime.
If $\exists n$ s.t. $\pi_i Y = 0$ $\forall i > n$, then
$Y\langle n\rangle \simeq * \Longrightarrow$

$$\hat{Y}_p = LY\langle n\rangle \simeq * \quad \forall p \Longrightarrow$$

$$Y \simeq *$$

2) If $\pi_2 Y$ is not finite, $\pi_2 Y = \oplus \mathbb{Z} \oplus$ finite

$\therefore$ $\exists$ bundle sequence

$$\prod S^1 \to W \to Y \to \prod \mathbb{C}P^\infty \quad \text{with:}$$

$\pi_2 W$ = finite, W finite dimensional

Apply 1) to W. $W \simeq *$, $\therefore$ $Y \simeq \prod \mathbb{C}P^\infty$, "