Step 2. $\mathrm{map}_*(B\mathbb{Z}/p, X) \simeq_{wk} *$ + $X$ $p$-complete
$G$ torsion free abelian
$\Rightarrow \mathrm{map}_*(K(G,2), X) \simeq_{wk} *$

Proof: exact $0 \to \mathbb{Z} \to \mathbb{Z}[\frac{1}{p}] \to \mathbb{Z}/p^\infty \to 0$

$\Rightarrow$ exact $0 \to G \to \mathbb{Z}[\frac{1}{p}] \otimes G \to \mathbb{Z}/p^\infty \otimes G \to 0$

$\Rightarrow$ fibration sequence
$$B(\mathbb{Z}[\tfrac{1}{p}] \otimes G) \to B(\mathbb{Z}/p^\infty \otimes G) \to K(G,2)$$

$B\mathbb{Z}[\frac{1}{p}] \otimes G$ has trivial mod $p$ homology

$\therefore$ $B(\mathbb{Z}/p^\infty \otimes G) \to K(G,2)$ is
mod $p$ homology isom.,
$\therefore$ local equivalence.

Step 3. ~~$\mathrm{map}_*$~~ $X$ $p$-complete, $H$ abelian torsion, all elements of order relatively prime to $p$ $\Rightarrow$
$\mathrm{map}_*(BH, X) \simeq_{wk} *$

Proof: $BH \to *$ is a mod $p$ homology isom.

Step 4. a) $\mathrm{map}_*(G, X) \simeq_{wk} * \Rightarrow \mathrm{map}_*(BG, X) \simeq_{wk} *$
b) $\mathrm{map}_*(BG, X) \simeq_{wk} * \simeq_{wk} \mathrm{map}_*(BH, X)$
+ exact sequence $1 \to H \to T \to G \to 1$
$\Rightarrow \mathrm{map}_*(BT, X) \simeq_{wk} *$