☆ Step 7. Y s.c. + $H_*(Y; \mathbb{Z}/p) = 0 \ \forall * >> 0$
+ $\pi_2 Y$ torsion $\Longrightarrow$

$$LY\langle n\rangle = \hat{Y}_p.$$

Proof: Claim: $Y\langle n\rangle \xrightarrow{i} Y \to \hat{Y}_p$ is

localization

1) i is a local equivalence: $Y\langle n\rangle \to Y = Y\langle 1\rangle$

fibrations $K(\pi_n, n) \to Y\langle n\rangle \to Y\langle n-1\rangle$
$\mathrm{map}(K(\pi_n, n), X) \simeq_{wk} * \quad \forall$ local X

$\Rightarrow Y\langle n\rangle \to Y\langle n-1\rangle$ local equiv.

2) $Y \to \hat{Y}_p$ mod p homology isom $\Rightarrow$

$Y \to \hat{Y}_p$ local equiv.

3) $\hat{Y}_p$ is local: $\hat{Y}_p$ p-complete +

$\mathrm{map}_*(B\mathbb{Z}/p, \hat{Y}_p) \simeq_{wk} *$