

Lecture 4. Applications of Exotic Localization and the Serre Conjecture

A. Applications to H-spaces

Preliminaries: $L = L_M = L \rightarrow M$

a. $X, Y \ M \text{ local} \Rightarrow X \times Y \ M \text{ local}:$
$$\text{map}(M, X \times Y) = \text{map}(M, X) \times \text{map}(M, Y)$$

b. $X \ M \text{ local}, C \text{ arbitrary} \Rightarrow \text{map}(C, X) \ M \text{ local}.$
$$\begin{aligned} \text{map}(M, \text{map}(C, X)) &= \text{map}(M \times C, X) \\ &= \text{map}(C, \text{map}(M, X)) = \text{map}(C, X) \end{aligned}$$

c. $X \ M \text{ local}, C \text{ arbitrary} \Rightarrow \text{map}_*(C, X) \ M \text{ local}$
similar to b. e.g. $X \ M \text{ local} \Rightarrow$
 $\Omega X \ M \text{ local}$

d. $A \sim B \ \text{loc equiv}, C \text{ arbitrary} \Rightarrow$
 $A \times C \sim B \times C \ \text{loc equiv}:$
$$\begin{aligned} \text{map}(A \times C, X) &= \text{map}(A, \text{map}(C, X)) \cong \\ &= \text{map}(B, \text{map}(C, X)) = \\ &= \text{map}(B \times C, X). \end{aligned}$$

e. $L(X \times Y) = LX \times LY:$

1) $LX \times LY$ is local

2) $X \times Y \rightarrow (LX) \times Y \rightarrow LX \times LY$
is a loc equiv.