

f. X H-space, multiplication $\mu: X \times X \rightarrow X$
 $\Rightarrow LX$ H-space, multiplication
 $L\mu: LX \times LX \rightarrow LX$

and: k -th power map $k: X \rightarrow X \Rightarrow$
 k -th power map $k = Lk: LX \rightarrow LX$

★ Cor: X s.c. finite complex, $\pi_2 X$ finite ~~implies~~
 ~~$X \langle n \rangle$~~ H-space for some $n \Rightarrow$
 \hat{X}_p H-space

ex: $S^2 \langle 2 \rangle = S^3$ is H-space
 but: $n > 1 \Rightarrow S^{2n} \langle k \rangle$ not H-space
 $\forall k$

Note: $X \rightarrow X_{(p)} \rightarrow \hat{X}_p$, $\pi_2 X$ finite $\forall p$

$$\Rightarrow X_{(p)} \xrightarrow{\cong} \hat{X}_p$$

e.g. $S^{2n+1} \langle 2n+1 \rangle_{(p)} = S^{2n+1} \langle 2n+1 \rangle_p$

Cohen-Moore-Neisendorfer summary:

Throughout, localize at a prime $p > 2$:

$\exists \pi: \Omega^2 S^{2n+1} \rightarrow S^{2n-1}$ such that:

$$\Sigma^2 \circ \pi = p: \Omega^2 S^{2n+1} \xrightarrow{\pi} S^{2n-1} \xrightarrow{\Sigma^2} \Omega^2 S^{2n+1}$$

commutes.

$p = p$ -th power