B. <u>Serre's conjecture</u>: X simply connected, $\pi_* X$ fin gen $\forall *$, $H_*(X; \mathbb{Z}/p) = 0$ for $* >> 0$,

p-tors $\pi_i X = 0$ for $i >> 0 \Longrightarrow$

$\bar{H}_*(X; \mathbb{Z}/p) = 0$ for all $* \geq 0$.

Step 1. <u>rationalization</u>: X simply connected,

nullify $M(\mathbb{Z}/q, 1)$ $\forall$ primes q, $X \mapsto LX = X \otimes \mathbb{Q}$

$$\bar{H}_*(X \otimes \mathbb{Q}) = \bar{H}_* X \otimes \mathbb{Q}$$
$$\pi_*(X \otimes \mathbb{Q}) = \pi_* X \otimes \mathbb{Q}.$$

<u>Cartan-Serre theorem</u>: X s.c. H-space $\Longrightarrow$

$$X \otimes \mathbb{Q} \simeq \prod_{n \geq 2} K(\pi_n, n)$$

Strong form: all rational k-invariants

$$k_i \otimes \mathbb{Q} = 0$$

Step 2. <u>Postnikov systems</u>, <u>k-invariants</u>, and <u>lifting problems</u>

X = s.c. space, $X = \varprojlim X_n$: $\pi_i X_n = \begin{cases} 0, & i > n \\ \pi_i X, & i \leq n \end{cases}$

fibration sequences:

$$K(\pi_i, i) \to X_i \to X_{i-1} \xrightarrow{k_i} K(\pi_i, i+1),$$
$$k_i \in H^{i+1}(X_{i-1}, \pi_i)$$

$$\begin{tikzcd}
 & & & & & K(\pi_4, 5) & K(\pi_3, 4) \\
X \arrow[r] & \cdots \arrow[r] & X_i \arrow[r] & \cdots \arrow[r] & X_4 \arrow[r] \arrow[u] & X_3 \arrow[r] \arrow[u, "k_4"] & X_2 = K(\pi_2, 2) \arrow[u, "k_3"]
\end{tikzcd}$$