$$0 = k_i \Rightarrow X_i \simeq X_{i-1} \times K(\pi_i, i)$$

lifting problem:

$$\begin{tikzcd} & X \arrow[r] & X_2 \\ & & Y \arrow[ul, dashed, "\exists f"] \arrow[u, "f_2"'] \end{tikzcd} \iff$$

$$\forall i, \exists \quad \begin{tikzcd} X_i \arrow[r] & X_{i-1} \arrow[r, "k_i"] & K(\pi_i, i+1) \\ & Y \arrow[ul, dashed, "\exists f_i"] \arrow[u, "f_{i-1}"'] \arrow[ur] & \end{tikzcd} \iff$$

$$k_i f_{i-1} = 0 \in H^{i+1}(Y, \pi_i).$$

Step 3. lift exists, if $H_*(Y;\mathbb{Z})$ free, $\pi_* X$ torsion free, X s.c. H-space.

Obstructions are 0:

$$k_i f_{i-1} \in H^{i+1}(Y;\pi_i)$$

$\quad\quad\quad\quad \| \leftarrow$ since $H_* Y$ free

$$\text{hom}(H_{i+1}Y, \pi_i)$$

monic $\quad\quad \downarrow \leftarrow$ since $\pi_i X$ torsion free

$$\text{hom}(H_{i+1}Y, \pi_i \otimes \mathbb{Q})$$

$$\quad\quad\quad\quad\quad\quad\quad\quad \|$$

$$0 = (k_i \otimes \mathbb{Q}) f_{i-1} \in H^{i+1}(Y;\pi_i \otimes \mathbb{Q})$$