

$$0 = k_i \Rightarrow X_i \cong X_{i-1} \times K(\pi_{i-1})$$

lifting problem: $X \rightarrow X_2$

$$\begin{array}{ccc} \exists f_1 \uparrow & & \uparrow f_2 \\ & \hat{=} & \\ & & Y \end{array} \iff$$

$$\forall i, \exists f_i \quad X_i \rightarrow X_{i-1} \xrightarrow{k_i} K(\pi_{i-1}) \iff$$

$$\begin{array}{ccc} \exists f_i \uparrow & & \uparrow f_{i-1} \\ & \hat{=} & \\ & & Y \end{array}$$

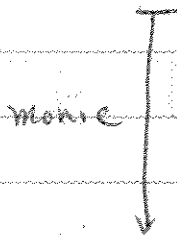
$$k_i \circ f_{i-1} = 0 \in H^{i+1}(Y, \pi_i)$$

★ Step 3. lift exists, if $H_*(Y; \mathbb{Z})$ free, $\pi_i X$ torsion free, X s.c. H-space.

Obstructions are 0:

$$k_i \circ f_{i-1} \in H^{i+1}(Y; \pi_i)$$

" \leftarrow since $H_* Y$ free



$$\text{hom}(H_{i+1} Y, \pi_i)$$

" \leftarrow since $\pi_i X$ torsion free

$$\text{hom}(H_{i+1} Y, \pi_i \otimes \mathbb{Q})$$

$$0 = (k_i \otimes \mathbb{Q}) \circ f_{i-1} \in H^{i+1}(Y; \pi_i \otimes \mathbb{Q})$$