# Lecture 1. Chain models for homotopy pullbacks

coalgebra model for a space or simplicial set $X$: given by a differential coalgebra $C$ for which $\exists$ a sequence of homology isom coalgebra maps

$$C_0 \leftarrow \rightarrow \leftarrow \rightarrow \cdots \leftarrow C_n$$

where $C_0 = C$, $C_n = C(X)$ = normalized chains with Alexander-Whitney diagonal

$$\Delta \langle v_0, \dots, v_n \rangle = \sum \langle v_0, \dots, v_i \rangle \otimes \langle v_i, \dots, v_n \rangle$$

and counit $\varepsilon: C(X) \to C(*) = R$.

formal space $X$: $HX$ is a chain model for $X$

e.g. $X$ = suspension $\Rightarrow$ $X$ is formal
$HX$ free over $R$ (not nec)

Proof: $X = x_0 * Y$, all $X$ primitive under A-W map: Write $C(X) = HX \oplus D$ as chain complexes: $HX \to C(X)$ is a homology isom coalg map.

e.g. $\exists$ homology isom coalg map

$$T(z_n) \xrightarrow{\theta} C(\Omega S^{n+1})$$

Proof: use assoc $\Omega$, $\therefore$ assoc $C(\Omega S^{n+1})$,