strong homological invariance

$$\left.\begin{array}{l} C \xrightarrow{\simeq} C_1 \\ M \xrightarrow{\simeq} M_1 \\ N \xrightarrow{\simeq} N_1 \end{array}\right. \Rightarrow \quad E(M) \square_C E(N) \xrightarrow{\simeq} E(M_1) \square_{C_1} E(N_1)$$

Proof: follows from Eilenberg-Moore spectral seq:

$$E^2 = E(HM) \square_{HC} E(HN) \Rightarrow E(M) \square_C E(N).$$

Eilenberg-Moore model for a homotopy pullback:

Let
$$\begin{tikzcd} E \arrow[r, "k"] \arrow[d, "h"'] & Z \arrow[d, "g"] \\ Y \arrow[r, "f"'] & X \end{tikzcd}$$
be a pullback with: g a fibration, X a 1-reduced simplicial set

Theorem: The composite
$$E \to Y \square_X Z \to E(Y) \square_X Z$$
is a homology isomorphism.