Proof: $F \to E \to Y$ and $F \to Z \to X$ are both oriented fibration sequences

Serre's computation: via the Serre filtration,

$$E^1(E) = Y \otimes HF \ , \quad E^1(Z) = X \otimes HF$$

Filter E, Z by Serre filtration

$E(Y)$ by total degree: $0 = d^0 = d^1$, $d^2 = d$

$E(Y) \,\square_X\, Z \subseteq E(Y) \otimes Z$ by product filtr.

A) $E \to E(Y) \,\square_X\, Z$ is filtr pres

B) Write $E(Y) = G \otimes X$: ~~ ~~ G, X free over R

$$E^0(E(Y) \,\square_X\, Z) = E(Y) \,\square_X\, Z = G \otimes X \,\square_X\, Z = G \otimes Z$$

$d^0 = 1 \otimes d^0$

C)

$$\begin{aligned} E^1(E(Y) \,\square_X\, Z) &= G \otimes E^1(Z) = G \otimes X \otimes HF \\ \uparrow \cong \qquad\qquad &= E(Y) \otimes HF \\ &\qquad \uparrow \cong \\ E^1(E) \qquad\qquad &= Y \otimes HF \end{aligned}$$

$\therefore$ $E \to E(Y) \,\square_X\, Z$ is a homology isom.